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APPLIED MATHEMATICS AND COMPUTATION
ELSEVIER Applied Mathematics and Computation 166 (2005) 457–463
www.elsevier.com/locate/amc

Analysis of three-dimensional grids: four- and five-point cubes

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Abstract

The estimation of quadratic coefficients on four or five data in prismatic array has been regarded as impossible. Estimates of these coefficients can be obtained by means of operational equations that use exponential representation. The coefficients are compared to the true values as obtained by Taylor expansions of the generating functions. If the data are monotonic, the accuracy of the estimations may be sufficient to interest experimentalists.

Keywords: Interpolation; Response surfaces; Quadratic coefficients; Curvature; Cubes

1. Introduction

The estimation of quadratic-term coefficients by means of eight or nine data in cubical array has recently been illustrated [1–3]. The expense of experimental work encourages the search for methods that yield these estimates but without

[1] Los Alamos National Library is operated by the University of California for the U.S. Department of Energy under Contract no. W-7405-ENG-36.

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2004.06.070

the need for eight or nine measurements. This paper illustrates the estimation of quadratic-term coefficients based on four or five measurements. The accuracy of the methods, combined with the economy that derives from reduced laboratory costs, may be sufficient to interest experimentalists.

2. The five-point cube

A recent manuscript has illustrated the treatment of five suitably-placed measurements in prismatic array [2]. The data are located at the cube vertices A, D, G, H and center point E as shown in Fig. 1. The method is based on representation of the five positive data by means of a power of a polynomial expression. Another method for treating the same array is based on an exponential representation. It turns on the solution of Eqs. (1)–(4). In these equations, capital letters A, D, G, H, and E represent numerical measurements taken at the same locations as illustrated by the cube shown in Fig. 1. Eqs. (1)–(4) are solved simultaneously for the parameters J, K, L, and T.

$$[(E-T)/(A-T)]-JKL=0, \tag{1}$$

$$[(E-T)/(D-T)]-L/(JK)=0, \tag{2}$$

$$[(E-T)/(H-T)]-J/(KL)=0, \tag{3}$$

$$[(E-T)/(G-T)]-K/(JL)=0. \tag{4}$$

The solutions generated by Eqs. (1)–(4) appear as a set. The member that is ordinarily chosen to represent the data is the one that contains only real, positive values of J, K, and L. Those values are substituted into Eq. (5). That equa-

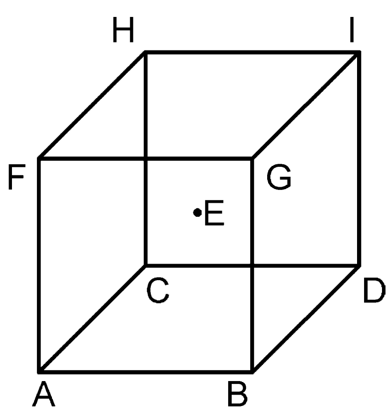

Fig. 1. Nine-point cube with center point E.