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Peristaltic transport of a compressible viscous liquid through a tapered pore

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Abstract

The axisymmetric peristaltic flow of a viscous compressible liquid through a tapered pore has been studied. This peristaltic flow is actually a result of the influence of ultrasonic radiation on the flow of a liquid through a porous medium which deforms the walls of the pores in the shape of travelling transversal waves exactly like peristaltic pumping [R.F. Ganiev, L.E. Ukrainskii, K.V. Frolov, Wave mechanism for the acceleration of a liquid flowing in capillaries and porous media, Sov. Phys. Dokl. 34 (1989) 519–521]. In this article, the wave amplitude is related to the power output of an acoustic source, while the wave speed is expressed in terms of the shear modulus of the porous medium and the dimensionless radius of the pore (variable radius/radius at inlet). A perturbation technique has been employed to analyze the problem where the amplitude ratio (wave amplitude/pore radius at inlet) is chosen as a parameter. In the second-order approximation a net flow induced by the travelling wave is found. The net flow rate is calculated for various values of the compressibility of the liquid, the Reynolds number and wavelength parameter. The calculations disclose that the compressibility of the liquid has a strong influence on the net flow induced. Furthermore, by a comparison with the flow induced by the pressure gradient in an oil reservoir, the net

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flow induced by a travelling wave cannot be neglected, although it is a second-order effect.

Keywords: Acoustics; Peristaltic transport; Travelling waves; Tapered tube; Compressible flow; Porous-media flow

1. Introduction

Many laboratory experiments have been made to investigate the effect of the ultrasonic radiation on the flow of a liquid through a porous medium, see for instance Chen [2], Cherskiy et al. [3] and Duhon [4]. They found that ultrasonic radiation increases the rate of flow of the liquid. Chen [2] shows also that the viscosity decrease caused by the ultrasonic energy, increases the flow rate only a part.

A theoretical explanation for the increase of the rate of flow of a liquid through a porous medium by ultrasonic radiation has been given by Ganiev et al. [5]. They propose that ultrasonic radiation generates travelling transversal waves at the pore walls in a porous medium which is identical to the peristaltic transport mechanism (with the pore approximated by a cylindrical tube of small radius). Various papers have been written about peristaltic transport, see for instance Shapiro et al. [9], Takabatake et al. [10], and Yin and Fung [11]. In particular, Yin and Fung [11] studied the flow of an incompressible viscous liquid through a cylindrical tube with its wall deformed in the shape of a traveling transversal wave. Later, Takabatake et al. [10] mention that the perturbation analysis of Yin and Fung [11] is only valid if the dimensionless parameters ε, α and Re satisfy $\varepsilon\alpha^2 Re \ll 1$, where ε is the amplitude ratio, α is the wavelength parameter and Re is the Reynolds number.

Aarts and Ooms [1] elaborated the mechanism proposed by Ganiev et al. [5] by representing the porous medium as a set of noninteracting, straight and circular capillaries in an elastic medium. The pores of any porous medium are in general irregular in shape [8], so it is more convincing to take any of them as a circular cylinder of changing cross section.

In this article we go the way as Aarts and Ooms [1] to study the axisymmetric peristaltic motion of a viscous compressible liquid through a flexible pore of changing cross section. The analysis has been carried out using a perturbation method, actually we extend the analysis of Yin and Fung [11] by taking the compressibility k of the liquid into account, also we extend the analysis of Aarts and Ooms [1] by taking the radius of the pore to vary linearly. We find that the dimensionless radius (variable radius/radius at inlet) of the pore affects the wave speed c of the peristaltic wave which means it also affects the compressibility parameter χ and the net flow induced $\langle Q\rangle$. Calculations disclose