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Exact solutions of compact and noncompact structures for the KP–BBM equation

Abdul-Majid Wazwaz

Department of Mathematics and Computer Science, Saint Xavier University, 3700 West 103rd Street, Chicago, IL 60655, USA

Abstract

The tanh and the sine–cosine methods are used for a reliable treatment of the (2 + 1) dimensional KP–BBM equation. Generalized forms of the KP–BBM equation are investigated as well. A variety of exact travelling wave solutions of distinct physical structures: compactons, solitons, solitary patterns and periodic solutions are formally derived.

Keywords: Periodic solutions; Compactons; Solitons; Benjamin–Bona–Mahony (BBM) equation; The (KP) equation; Sine–cosine method; The tanh method

1. Introduction

The Benjamin–Bona–Mahony (BBM) equation

$$u_t + u_x + uu_x - u_{xxt} = 0 \tag{1}$$

E-mail address: wazwaz@sxu.edu

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has been proposed as a model for propagation of long waves [1] where nonlinear dispersion is incorporated.

The balance between the nonlinear convection term uu_x and the dispersion effect term u_{xxx} in the spatially one-dimensional KdV equation

$$u_t + a(u^2)_x + u_{xxx} = 0 \tag{2}$$

gives rise to solitons. The term soliton coined by Zabusky and Kruskal [2] who found particle like waves which retained their shapes and velocities after collisions. The KdV equation is a model that governs the one-dimensional propagation of small amplitude, weakly dispersive waves [2–12]. However, the delicate interaction between nonlinear convection $(u^n)_x$ with genuine nonlinear dispersion $(u^n)_{xxx}$ in the $K(n,n)$ equation

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0 \tag{3}$$

generates solitary waves with compact support that are termed *compactons* [6]. Unlike solitons that narrows as the amplitude increases, the compacton's width is independent of the amplitude.

A variety of useful methods: the tanh–sech method and the extended tanh method [6–13], the trial function [14,15], Backlund transformation, the inverse scattering method, bilinear transformation, the sine–cosine method [16–24], pseudo spectral method [6], and the homogeneous balance method were used to investigate nonlinear dispersive and dissipative problems.

Two well-known two-dimensional generalizations of the KdV equations were developed, namely the Kadomtsov–Petviashvili (KP) equation [4], and the Zakharov–Kuznetsov (ZK) [3] given by

$$\{u_t + auu_x + u_{xxx}\}_x + u_{yy} = 0 \tag{4}$$

and

$$u_t + auu_x + (\nabla^2 u)_x = 0, \tag{5}$$

respectively, where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the isotropic Laplacian. In other words, the ZK equation is given by

$$u_t + auu_x + (u_{xx} + u_{yy} + u_{zz})_x = 0. \tag{6}$$

The goal of this work is to study the Benjamin–Bona–Mahony equation and two of its variants formulated in the KP sense. In other words, we will examine the nonlinear (2 + 1) dimensional KP–BBM problem

$$(u_t + u_x - a(u^2)_x - bu_{xxt})_x + ru_{yy} = 0, \tag{7}$$

a generalized form of the KP–BBM

$$(u_t + u_x - a(u^n)_x - bu_{xxt})_x + ru_{yy} = 0, \quad n > 1, \tag{8}$$