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Approximate computation of eigenvalues with Chebyshev collocation method

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Abstract

In this study, Chebyshev collocation method is investigated for the approximate computation of higher Sturm–Liouville eigenvalues by a truncated Chebyshev series. Using the Chebyshev collocation points, this method transform the Sturm–Liouville problems and given boundary conditions to matrix equation. By solving the algebraic equation system, the approximate eigenvalues can be computed. Hence by using asymptotic correction technique, corrected eigenvalues can be obtained.

Keywords: Eigenvalue problem; Collocation method; Chebyshev series

1. Introduction

The concept of an eigenvalue problem is rather important for both in pure and applied mathematics, a physical system, such as a pendulum, a vibrating and rotating shaft. All these physical systems are connected with eigenpairs of the system.

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A general Sturm–Liouville problem can be written as the following differential equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(p(x)\frac{\mathrm{d}u}{\mathrm{d}x}\right)+(r(x)\lambda-q_1(x))u=0.$$

This equation can be reduced to the canonical Liouville normal form,

$$u''+(\lambda-q(x))u=0.$$

In this study, the Liouville normal form was investigated. If it is difficult to solve the Sturm–Liouville problems or there are no exact solutions of Sturm–Liouville problems, they can be solved by various approximate methods.

For the solution of the eigenvalue problem, some studies have been carried out. Fox and Parker [8] used Chebyshev series to solve differential eigenvalue problems. Pain, de Hoog and Anderson [10] showed that in the case of the second order centered finite difference method with uniform mesh, the error, when $q(x)$ is constant (in the Liouville normal form), has the same asymptotic form (for $k\to\infty$) as the error for general $q(x)$. In the study, they gave corrected finite difference approximation.

Fix [7] investigated the following Sturm–Liouville problem for general $q(x)$.

$$\left.\begin{array}{l} u''+(\lambda-q(x))u=0,\quad 0\leqslant x\leqslant \pi \\ u'(0)-\alpha u(0)=u'(\pi)-\beta u(\pi)=0 \\ \text{or} \\ u(0)=u(\pi)=0 \end{array}\right\}. \tag{1}$$

In order to study the asymptotic behavior of eigenvalues for large k, he used the function $\Phi(x,\lambda)$, the modifier Prüfer phase, which is defined for any given solution $u(x,\lambda)$ of Eq. (1) by the equation

$$\tan(\Phi)=(\lambda-q(x))^{\frac{1}{2}}\frac{u'}{u}$$

and he found recurrence formulas to obtain eigenvalues of Eq. (1) for general $q(x)$.

Andrew and Paine [3] improved the results of Numerov's method with asymptotic correction technique. Vanden Berghe and De Meyer [6] have developed special two step methods producing very accurate results. Ghelardnoi [9] investigated the approximations of Sturm–Liouville eigenvalues using some linear multistep methods, called Boundary Value Methods and correction technique of Andrew–Paine and Paine et al. [10] is extended to these methods.

By using the following Liouville normal form

$$\begin{aligned} &u''+(\lambda-q(x))u=0,\\ &u(0)=u(\pi)=0, \end{aligned}$$

the asymptotic correction technique which [3,10] shown can be outlined as follows.