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Connectivity graphs as models of local interactions ☆

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Abstract

In this paper, we study graphs that arise from certain sensory and communication limitations on the local interactions in multi-agent systems. In particular, we show that the set of graphs that can represent formations corresponds to a proper subset of all graphs and we denote such graphs as connectivity graphs. These graphs have a special structure that allows them to be composed from a small number of atomic generators using a certain kind of graph amalgamation. This structure moreover allows us to give connectivity graphs a topological characterization in terms of their simplicial complexes. Finally, we outline some applications of this topological characterization to the construction of decentralized algorithms.

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1. Introduction

The problem of coordinating multiple mobile robots is one in which a finite representation of the configuration space appears naturally, namely by using graph-theoretic models for describing the local interactions in the formation. In other words, graph-based models can serve as a bridge between the continuous and the discrete when trying to manage the design-complexity associated with formation control problems. Notable results along these lines have been presented in [1–3], where graphs are used for modelling what neighboring robots a given robot can communicate with. In [4], the idea is to represent the desired formation as a graph, and then produce formations from free agents such that the appropriate links are formed between them. In [5], such graph-models were successfully used for showing how the use of nearest neighbor average heading rules asymptotically produced subgraphs in which all the robots maintained the same heading.

The conclusion to be drawn from these research efforts is that a number of questions can be answered in a natural way by abstracting away the continuous dynamics of the individual agents. In particular, if the existence of an edge between two agents corresponds to certain geometric relations such as spatial closeness, then the graphs are really dynamic in the sense that the time-driven evolution of the configuration space may result in the production of a new graph. Hence, by introducing a graph-based representation of the formation, one has in fact arrived at a hybrid dynamic system. Furthermore, information can be allowed to flow along the edges of the graph, thus resulting in a change of the time-driven dynamics based on events at the graph-level, which further stresses the hybrid nature of this construction.

In algebraic topology literature, the connectivity graphs appear as *nerves* of certain covers for sets. The concept of giving a topological representation to the union of a collection of sets is not new. Eduard Čech introduced his Čech-homology theory in 1932, which deals with such representations [10]. This work however remained un-noticed in applied sciences till it re-appeared in [6,7] in the 1990s. In [6], unions of disks were considered, the nerve of which are exactly connectivity graphs and [7] describes some *kinetic data structures* that capture the changes in topology of dynamic connectivity graphs. However, it should be noted that this work differs from those mentioned above in many ways. Although Čech homology fully describes the topological shape of the collection of sets (or its dual, the connectivity graph), it is difficult to compute. The framework of connectivity graphs described below is easy to implement. More importantly, this framework is tailored specifically for implementation on a distributed (and decentralized) system of agents.

In this paper we show that the graphs that can represent formations do in fact correspond to a proper subset of all graphs, denoted by the set of connectivity graphs (Section 2), and that it is possible to give a topological character-