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An acceleration of iterative processes for solving nonlinear equations

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Abstract

The purpose of the paper is to present a method for acceleration of iterative processes for solving a nonlinear equation. This approach is generalized for the case of multiple roots when the multiplicity rate is preliminarily known. Thus some new methods are obtained. The convergence analysis of the presented method and numerical examples are given.

Keywords: Iterative method; Simultaneous root finding; Order of convergence; Multi-point iterative process

1. Introduction

The acceleration of convergence has been an active field of research in numerical analysis. There are two types of root-finding algorithms for solving nonlinear equations (algebraic equations), one of which is referred to a *single-root* algorithm and another one is a *simultaneous* one. The single-root algorithm converges to one root, like Newton's or Bairstow's methods. The

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simultaneous algorithm improves all the approximations, simultaneously, which converge to the corresponding zeros.

Very often, when we solve some practical problems, it may happen that the chosen algorithm very slow converges to the sought root or roots. In such cases there are two possibilities: first we can give up the chosen method and try to find another one, for which there is a quicker convergence; the other one is to accelerate the first one.

In this article we consider an approach for acceleration of iterative processes for solving nonlinear equations. In [7] is examined a method for acceleration of algorithms, in the case when we seek of one and simple root. Here we will generalize this method for more common cases.

In Section 2 we modify the aforementioned method for the case of approximation a multiply root of nonlinear equation. Analogically, in Section 3 the approach is proved for the case of simultaneous root finding processes for multiple roots of algebraic polynomials. Some specific examples of iterative formulae are presented in Section 4. The results of numerical experiments are included in Section 5.

2. Acceleration of iterative processes for approximation of only one root of a nonlinear equation

We consider the nonlinear equation:

$$f(x) = 0. \tag{1}$$

Let us have the iterative function of the form

$$y = x - f(x)\varphi(x) = x - v(x) \tag{2}$$

for approximation of α—one of the roots of Eq. (1).

In the case when the root α is simple in [7] it is examined a method for acceleration of the function (2), namely it is effective the following:

Theorem 1. *Let $\varphi(x), f(x) \in C^k[a,b]$, $\varphi(x)\cdot f'(x) \neq 0$ for every $x \in [a,b]$ and α is a root of the equation $f(x) = 0$, located in the interval (a,b). If the iterative function*

$$y = x - \varphi(x)f(x)$$

for computing of α has a convergence order k, i.e.

$$y'(\alpha) = y''(\alpha) = \cdots = y^{(k-1)}(\alpha) = 0, \quad y^{(k)}(\alpha) \neq 0$$

and the function

$$\psi(x) = \begin{cases} (1 - y'(\alpha))^{-1}(1 + O(\varepsilon)) & \text{for } k = 1, \\ 1 + \frac{\varepsilon^{k-1}y^{(k)}(\alpha)}{k!} + O(\varepsilon^k) & \text{for } k > 1, \end{cases} \tag{3}$$

where $\varepsilon = x - \alpha$, $x \in [a,b]$ and x is sufficiently close to α. Then: