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Representation of the exact solution for infinite system of linear equations

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Abstract

In this paper, representation of the exact solution for the infinite system of linear equations is given. And we give a sufficient and necessary condition for the infinite system of linear equations which has solutions.

Keywords: Infinite system of linear equations; Reproducing kernel; Operator equation

1. Introduction

In 1962, the convergency of iteration method for regular infinite system of equations

$$X - \lambda AX = b \tag{1}$$

was discussed by Kontorovich in Ref. [1], where $A = (a_{ij})_{i,j=1}^{\infty}$ is an infinite matrix satisfying condition

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$$\sum_{i,j=1}^{\infty} a_{ij}^2 < \infty \tag{2}$$

and $X = (x_1, x_2, \ldots)^{\mathrm{T}} \in l^2, b = (b_1, b_2, \ldots)^{\mathrm{T}} \in l^2$, and A satisfies

$$1 - \sum_{i,j=1}^{\infty} |a_{ij}| \geqslant 0 \quad \text{(regular condition)}.$$

In Ref. [2], he demonstrated the convergency of the intercept equation

$$X_n - \lambda A_n X_n = b_n \tag{3}$$

while condition (2) holds, where $A_n = (a_{ij})_{i,j=1}^{n}$, $X_n = (x_{n1}, x_{n2}, \ldots, x_{nn})^{\mathrm{T}}$, and $b_n = (b_{n1}, b_{n2}, \ldots, b_{nn})^{\mathrm{T}}$. But since then, one has seldom seen any discussions about the infinite system of linear equations in correlative literatures for a long time. Up until the year 2002, we found the paper [3], but it merely discussed some special issues on structured and infinite systems of linear equations.

This paper will present a representation of the exact solution for the infinite system of linear equations

$$Ay = b \tag{4}$$

and it is given by the following steps:

1. Suppose A is a bounded linear operator from l^2 onto l^2.
2. Establish a one-to-one mapping $\rho : l^2 \to W_2^1[0,1]$, where the definition of the space $W_2^1[0,1]$ see [4].
3. Transform equation (4) into equation
$$Ku = f, \tag{5}$$
where, $u \in W_2^1[0,1], f = \rho b, K = \rho A \rho^{-1}$. (5) is an operator equation on $W_2^1[0,1]$.
4. Give the representation of the exact solution for Eq. (5).
5. Finally, represent the exact solution for the infinite system of linear equations (4) by $y = \rho^{-1} u$.

Besides, this paper gives a sufficient and necessary condition for the infinite system of linear equations which has solutions.

Since exact solution of (5) is given in the form of series, truncate the series, we obtain an approximate solution u_n of (5). Then we can get an approximate solution ${}_n y$ of (4). This approximate solution is explicit representation, and guarantees convergency.

The definition of space $W_2^1[0,1]$ was given in Ref. [4]:

Definition 1.1. $W_2^1[0,1] = \{u(x) | u(x)$ is an absolute continuous real-valued function on interval $[0,1]$ and $u'(x) \in L^2[0,1]\}$. And the inner product in $W_2^1[0,1]$ is provided by