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The Choi–Saigo–Srivastava integral operator and a class of analytic functions

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Abstract

Let $\mathscr{A}$ be the class of the normalized analytic functions in the unit disk $\mathbb{U}$, and $\mathscr{K}(\alpha)$ denote the subclass of $\mathscr{A}$ consisting of the convex functions of order α in $\mathbb{U}$. It is known that the operator $f_{\lambda,\mu}$ was defined such that $f_\lambda * f_{\lambda,\mu} = z/(1-z)^\mu\,(\mu > 0)$, where $*$ denotes the Hadamard product and $f_\lambda = z/(1-z)^{\lambda+1}\,(\lambda > -1)$. The Choi–Saigo–Srivastava integral operator $\mathscr{I}_{\lambda,\mu}$ was defined such that $\mathscr{I}_{\lambda,\mu}f = f_{\lambda,\mu} * f$. By using the operator $\mathscr{I}_{\lambda,\mu}$, we define the class $\mathscr{K}(\lambda,\mu)(\alpha) = \{f \in \mathscr{A} | \mathscr{I}_{\lambda,\mu}f \in \mathscr{K}(\alpha)\}$. In this paper, we study various inclusion properties of this class, some distortion theorems and coefficient inequalities. We have also provided some well-known results as corollaries of our theorems.

Keywords: Distortion theorems; Analytic functions; Choi–Saigo–Srivastava integral operator; Univalent functions; Convex functions; Hypergeometric function

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1. Introduction and definitions

Let $\mathscr{A}$ denote the class of functions $f(z)$ *normalized* by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are *analytic* in the *open* unit disk

$$\mathbb{U} := \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Also, given $\alpha < 1$, let $\mathscr{S}$, $\mathscr{S}^*(\alpha)$ and $\mathscr{K}(\alpha)$ denote the subclasses of $\mathscr{A}$ consisting of functions which are, respectively, *univalent*, *starlike of order* α and *convex of order* α in $\mathbb{U}$ (see, for details, [4], [5] and [8]). In particular, the class

$$\mathscr{S}^*(0) = \mathscr{S}^* \text{ and } \mathscr{K}(0) = \mathscr{K}$$

are the well-known classes of starlike and convex functions in $\mathbb{U}$, respectively.

A function $f(z) \in \mathscr{A}$ is said to be in the class of *prestarlike functions of order* α ($\alpha < 1$), denoted by $\mathscr{P}(\alpha)$ [7], if

$$\frac{z}{(1-z)^{2(1-\alpha)}} * f(z) \in S^*(\alpha), \quad z \in \mathbb{U}, \tag{1}$$

where "$*$" denotes the Hadamard product (or convolution). In [7], it is shown that $\mathscr{P}(\alpha) \subset \mathscr{P}(\beta)$ for $\alpha \leqslant \beta < 1$. Also it is easy to show that $\mathscr{P}(\frac{1}{2}) = \mathscr{S}^*(\frac{1}{2})$, $\mathscr{P}(0) = \mathscr{K}$.

Almost two decades ago, Carlson and Shaffer [2] defined a linear operator $\mathscr{L}(a,c) : \mathscr{A} \to \mathscr{A}$ by

$$\mathscr{L}(a,c)f(z) := \varphi(a,c;z) * f(z) \quad (f(z) \in \mathscr{A}), \tag{2}$$

where

$$\varphi(a,c;z) := \sum_{k=0}^{\infty} \frac{(a)_k}{(c)_k} z^{k+1} (z \in \mathbb{U}; c \notin \mathbb{Z}_0^- := \{0,-1,-2,\ldots\}) \tag{3}$$

and $(\lambda)_k$ denotes the Pochhammer symbol given, in terms of Gamma functions, by

$$(\lambda)_k := \frac{\Gamma(\lambda+k)}{\Gamma(\lambda)} = \begin{cases} 1 & (k=0), \\ \lambda(\lambda+1)\ldots(\lambda+k-1) & (k \in \mathbb{N}). \end{cases}$$

The operator $\mathscr{L}(a,c)$ maps $\mathscr{A}$ onto itself and is continuous on $\mathscr{A}$ (see [2]). In addition, for $c > a > 0$, $\mathscr{L}(a,c)$ has the integral representation

$$\mathscr{L}(a,c)f(z) = \int_0^1 \frac{f(uz)}{u} \mathrm{d}\mu(a,c-a)(u), \tag{4}$$