Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 165 (2005) 647–668
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Schwarz alternating algorithms for a convection–diffusion problem

Igor Boglaev

Institute of Fundamental Sciences, Massey University, Private Bag 11-222, Palmerston North, New Zealand

Abstract

This paper deals with iterative domain decomposition algorithms for solving a singularly perturbed convection–diffusion problem. The algorithms are based on Schwarz alternating procedure applied to a discrete approximations of the continuous problem. Convergence properties of the algorithms are established, and their parallel implementations are discussed. Numerical results for a test singularly perturbed problem are presented.

Keywords: Singularly perturbed convection–diffusion problem; Domain decomposition method; Parallel computing

1. Introduction

We are interested in iterative domain decomposition methods for solving the convection–diffusion problem with regular boundary layers

E-mail address: i.boglaev@massey.ac.nz

0096-3003/$ - see front matter
doi:10.1016/j.amc.2004.04.100

$$\begin{aligned} & -\varepsilon\Delta u + b_1\frac{\partial u}{\partial x} + b_2\frac{\partial u}{\partial y} + cu = f, \quad u = g \text{ on } \partial\Omega, \\ & b_1 \geqslant \beta_1 > 0, \quad b_2 \geqslant \beta_2 > 0, \quad c \geqslant c_* > 0, \quad \text{on } \bar{\Omega}, \end{aligned} \tag{1}$$

where $\Omega = \{P = (x, y) : 0 < x < 1, 0 < y < 1\}$, ε is a small positive parameter, β_1, β_2 and c_* are constants and $\partial\Omega$ is the boundary of Ω. If the data of the problem are sufficiently smooth, then under suitable continuity and compatibility conditions on the data, a unique solution $u(P)$ of (1) exists (see [1] for details). Furthermore, for $\varepsilon \ll 1$, problem (1) is singularly perturbed and characterized by boundary layers (i.e., regions with rapid change of the solution) of width $O(\varepsilon|\ln\varepsilon|)$ at $x = 1$ and $y = 1$ (see [2] for details).

Iterative domain decomposition algorithms based on Schwarz-type alternating procedures for solving singularly perturbed problems have received much attention for their potential as efficient algorithms for parallel computing (cf., for example, [3–7] and references cited there).

In [5], for solving problem (1), the classical Schwarz alternating method and some variants of it were analysed. In the case of domain decomposition into two subdomains, a convergence rate for the continuous problem (i.e. without resort to discretization in subdomains) as a function of the perturbation parameter ε and an amount of overlap between two subdomains was studied. It was shown that the Schwarz-type iterates $\{u^{(n)}\}$ converge to the exact solution u at the rate

$$\max_{\bar{\Omega}} | u^{(n+1)} - u | \leqslant \exp(-\alpha d\varepsilon^{-1}) \max_{\bar{\Omega}} | u^{(n)} - u |,$$

where $d \geqslant 0$ measures the overlap between the two subdomains, and $\alpha > 0$ is a constant independent of ε.

In [3], on the basis of asymptotic criteria, representations of optimal interface positions for the Schwarz alternating procedure were derived. For one dimension version of problem (1), in the case of domain decomposition into the two subdomains $[0, x_1]$, $[x_2, 1]$, $x_1 > x_2$, the interface positions x_1, x_2 from [3] are of order $O(\varepsilon|\ln\varepsilon|)$. If the number of mesh points in each subdomain is the same, N, then this interface condition is satisfied when N is of order $O(1/(\varepsilon|\ln\varepsilon|))$. Since the number of mesh points depends inversely on the perturbation parameter, then, in general, this approach leads to a nonuniform (in the perturbation parameter) convergent domain decomposition procedure.

In [7], a two-level iterative domain decomposition method with overlapping vertical strips was introduced. The iterative method from [7] consists of the two iterative processes: outer iterations and inner iterations. One outer iteration represents computing difference problems on the overlapping subdomains in serial, starting from the first left subdomain and finishing off on the last right subdomain (according to upwind error propagation). Thus, the multiplicative Schwarz method is the outer part of the algorithm. At the level of the inner