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A viscoelastic model of the brain parenchyma with pulsatile ventricular pressure

S. Sivaloganathan [a,*], M. Stastna [a], G. Tenti [a], J.M. Drake [b]

[a] *Department of Applied Mathematics, University of Waterloo, Waterloo, Ont., Canada N2L 3G1*
[b] *Division of Neurosurgery, Hospital for Sick Children, University of Toronto, Toronto, Ont., Canada M5G 1X8*

Abstract

In this paper, we present an extension of the model developed by Sivaloganathan et al. [Appl. Math. and Comput., to appear], which is of more physical relevance. We obtain explicit solutions for the displacement and stresses, and show how the mechanical parameters, that appear in the constitutive equation for the viscoelastic solid, can be calculated from data obtained from dynamic load experiments. Finally, we solve the boundary value problems corresponding to the case of adult hydrocephalus, as well as the more general case where both dilatational and deviatoric responses are assumed to be viscoelastic.

* Corresponding author.
E-mail address: ssivalog@sumathi.uwaterloo.ca (S. Sivaloganathan).

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1. A viscoelastic model of the brain parenchyma with pulsatile ventricular pressure

The classical theory of consolidation has been used in Tenti et al. [1] to study some of the biomechanical properties of the brain which are of importance in hydrocephalus research. A complementary approach which is equally of relevance in the study and understanding of hydrocephalus is the determination of the viscoelastic behaviour of the brain parenchyma and this has been examined in Sivaloganathan et al. [2].

The simplified model used in [1] and [2] represents the brain as a thick-walled tethered cylindrical tube so that the strain distribution is planar. The inside wall of the tube is kept at a constant pressure p_i while the outside wall is kept at a lower constant pressure p_0.

In this paper we consider an extension of the work presented in [2] which is possibly of more physical relevance. We replace the constant pressure at the inner wall by a pulsatile pressure. Physically, the beating of the heart is felt as a periodic pulse within the ventricular cavity. An analysis of this kind allows for the distinct possibility of determining relevant mechanical parameters from the data obtained from dynamic load experiments many of which have been reported in the literature (see [3–5]). We follow standard practice (see [6,7]) in assuming that the deviatoric parts of the stress and strain tensors behave in accordance with the standard viscoelastic models, whilst the dilatational part still obeys Hooke's law. This simplifies the mathematics, and the number of mechanically significant parameters is kept to a reasonable minimum. However it is possible with a little further effort to model both deviatoric and dilatational responses as viscoelastic, and to solve the problem where the displacement on the outer boundary is zero (this corresponds to the case of adult hydrocephalus where the skull or outer boundary is rigid).

2. Mathematical formulation

For a review of the basic elements of linear viscoelasticity theory, we refer the reader to [6] and [8]. An important question to be considered in modelling the brain parenchyma concerns the most appropriate choice of viscoelastic model that would realistically model the behaviour of the brain tissue. Clearly, the Maxwell and Kelvin–Voigt models are unsuitable in light of the objections raised in [2], and thus we use the "standard solid" as a more appropriate model than the Maxwell or Kelvin–Voigt model. The constitutive equation for the standard solid is given by (see [2])

$$\left(\frac{1}{E_1}+\frac{1}{E_2}\right)\sigma+\frac{\eta}{E_1E_2}\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\varepsilon+\frac{\eta}{E_2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}. \tag{1}$$