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ELSEVIER Applied Mathematics and Computation 164 (2005) 45–69

APPLIED MATHEMATICS AND COMPUTATION

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Log-barrier method for two-stage quadratic stochastic programming ☆

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Abstract

In this paper we present a log-barrier method for solving two-stage quadratic stochastic programs. The mathematical model considered here can be used to present several real world applications, including financial and production planning problems. We discuss fundamental properties associated with the proposed algorithm and analyze the convergence and complexity of the algorithm.

Keywords: Two-stage stochastic programming; Log-barrier method; Complexity; Polynomial algorithm

1. Introduction

In this paper, we study the following two-stage quadratic stochastic programs:

☆ This work was supported by Dongseo University, "Dongseo Frontier Project" Research Fund of 2002. This work was done when the author visited the Department of Computing and Software, McMaster University. The visit was also supported by McMaster University.
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0096-3003/$ - see front matter
doi:10.1016/j.amc.2004.04.095

$$\begin{aligned}\text{(QSP)}\quad \min\quad & c^{\mathrm{T}}x + \frac{1}{2}x^{\mathrm{T}}Px + \rho(x)\\ \text{s.t.}\quad & Ax = b,\\ & x \geqslant 0,\end{aligned} \tag{1}$$

where $P \in \Re^{n\times n}$ is symmetric positive semidefinite, $A \in \Re^{s\times n}$, $c \in \Re^{n}$ and $b \in \Re^{s}$, and $\rho(x)$ is the recourse function defined by

$$\rho(x) = E[\rho(x, \tilde{\xi})], \tag{2}$$

where, for each realization $\xi \in \Xi$

$$\begin{aligned}\rho(x,\xi) := \min\quad & q(\xi)^{\mathrm{T}}y(\xi) + \frac{1}{2}y(\xi)^{\mathrm{T}}D(\xi)y(\xi)\\ \text{s.t.}\quad & T(\xi)x + W(\xi)y(\xi) = h(\xi),\\ & y(\xi) \geqslant 0.\end{aligned} \tag{3}$$

Throughout this paper, E stands for expectation with respect to the random variable $\tilde{\xi}$, Ξ is the support of $\tilde{\xi}$. The problem (1) in the first stage is called the master problem where all the cost coefficient matrix P and vector c, the constraint matrix A and vector b are assumed to be deterministic. The recourse problem (3) in the second stage is called a subproblem where the associated cost coefficient vector $q(\xi) \in \Re^{m}$, and matrix $D(\xi) \in \Re^{m\times m}$ is symmetric positive semidefinite the recourse matrix, $W(\xi) \in \Re^{l\times m}$ and demand vector $h(\xi) \in \Re^{l}$, and the technology matrix $T(\xi) \in \Re^{l\times n}$ are all dependent on the random vector $\xi \in \Xi$. Further, $E[\rho(x, \tilde{\xi})]$ denotes the expected value of minimum extra costs based on first-stage decision and random events. It is worthwhile mentioning that the usual origin of the constraint equations in (3) is the desire to satisfy the condition $T(\xi)x - h(\xi) = 0$. However, because of the randomness of both T and h, this requirement can hardly be satisfied in general. Thus W_y is introduced to represent the "discrepancy".

Stochastic programs with recourse assign a probability distribution to uncertain parameters, design "recourse", and minimize the cost of the master decision plus the expected cost of the recourse decision. Stochastic programming provides a powerful tool to treat problems under uncertainty. In particular quadratic stochastic programs have a special role in mathematical programming with important application in finance and risk management. For instance two-stage financial planning problems that use quadratic risk measures can be modelled QSPs [7]. For a basic introduction to stochastic programming, its application and numerical methods, we refer the reader to [2].

Note that by the convexity of $\rho(\cdot, \xi)$ [3], the objective function in (1) is convex, and hence, the two-stage stochastic programming problem (1)–(3) is essentially a convex nonlinear programming problem. However, in general it is very