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Numerical solution of linear integral equations by using Sinc–collocation method

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Abstract

A collocation procedure is developed for linear Fredholm integral equations of the second kind, using Sinc basis functions. The convergence rate of the method is $O(e^{-k\sqrt{M}})$. Numerical results are included to confirm the efficiency and accuracy of the method.

Keywords: Sinc function; Collocation method; Fredholm integral equation

1. Introduction

In this paper, a Sinc–collocation procedure is developed for the numerical solution of the Fredholm integral equation of the second kind of the form:

$$u(x) = f(x) + \lambda \int_{\Gamma} k(x,t)u(t)\mathrm{d}t, \quad x,t \in \Gamma = [a,b], \tag{1}$$

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where the parameter λ, $k(x,y)$ and $f(x)$ are known functions and $u(x)$ is the solution to be determind. The Sinc collocation method for the initial value problems using the globally defined Sinc basis functions was proposed by Carlson et al. [1], the Sinc–Galerkin method for second order differential equations are given by Stenger [2], the Sinc–Galerkin method of Stenger, extended to numerical solution of fourth-order ordinary differential equations by Smith et al. [3], the Sinc–collocation procedures for the eigenvalue problems have been given in [4,5] and for the two point boundary value problems in [6,7]. Extensive summary of procedures and properties of Sinc Approximation can be found in [8]. In this paper a collocation procedure is developed for the Fredholm linear integral equation (1). First of all in Section 2 by using [8] briefly given the definitions and the theorems which are useful. In Section 3 the Sinc–collocation procedure associated four various boundary conditions for approximate the solution of (1) are given in Section 4. Numerical solution of four different examples are given to confirm the efficiency and accuracy of the present method.

2. Sinc interpolation

The Sinc function is defined on the whole real line by

$$\mathrm{Sinc}(x) = \begin{cases} \frac{\sin(\pi x)}{\pi x}, & x \neq 0; \\ 1, & x = 0. \end{cases} \tag{2}$$

For any $h > 0$, the translated Sinc functions with evenly spaced nodes are given as

$$S(j,h)(x) = \mathrm{Sinc}\left(\frac{x - jh}{h}\right), \quad j = 0, \pm 1, \pm 2, \ldots \tag{3}$$

which is called the jth Sinc function. The Sinc function for the interpolating points $x_k = kh$ is given by

$$S(j,h)(kh) = \delta_{jk}^{(0)} = \begin{cases} 1, & k = j; \\ 0, & k \neq j. \end{cases} \tag{4}$$

If u is defined on the real line, then for $h > 0$ the series

$$C(u,h)(x) = \sum_{j=-\infty}^{\infty} u(jh)\mathrm{Sinc}\left(\frac{x - jh}{h}\right) \tag{5}$$

is called the Whittaker cardinal expansion of u whenever this series converges, u is approximated by using the finite number of terms in (5). For positive integer N, we defined