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An approach to VaR for capital markets with Gaussian mixture

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Abstract

An approach to VaR (value-at-risk) for capital markets is proposed with Gaussian mixture. Considering the impacts of the components in a Gaussian mixture, an approach to VaR for capital markets is proposed to describe risk structure in capital markets. This approach can be programmed in parallel. Empirical computation of VaR for China securities markets and the Forex markets are provided to demonstrate the proposed method.

Keywords: Gaussian mixture; Value-at-risk; VaR; Parallel Computation

1. Introduction

The computational framework for value-at-risk (VaR) is a useful methodology for estimating the exposure of a given portfolio of securities to different

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kinds of risk inherent in financial environment. One driving force behind the popularity of this technique is the release to the studies of many researchers and the documents of JP Morgan and Basle Committee on Banking Supervision [1,2]. Another one is so-called random walks hypothesis about capital markets [3]. However, empirical evidence showed that the returns are actually fat tailed and that there exists non-random walks in capital markets and thus suggested that the assumption that returns of financial assets are normally distributed is inappropriate [4,5]. VaR calculated under the normal assumption underestimates the actual risk [6–9]. Zangari treated two components in a Gaussian mixture to fit the fat of financial assets [6]. Venkataraman provided an estimation techniques for value-at-risk in a mixture of normal distributions [7]. Hull and White suggested to use alternative distributions, such as a mixture of two normal distributions, to model the return of financial assets [8]. Li made use of statistics such as volatility, skewness and kurtosis to capture the extreme tail [9]. It is seen that these methods paid attention to few components in a Gaussian mixture. However, empirical studies showed that the number of components is greater than two in a Gaussian mixture for the returns of financial assets [10–12].

In this paper, an approach to VaR for capital markets with Gaussian mixture takes into account of not only non-normal distribution but also the impacts of the components in a Gaussian mixture. In Section 2, we propose an approach to VaR for capital markets with Gaussian mixture that uses lots of the components in a Gaussian mixture to describe risk structure in capital markets. In Section 3, we provide empirical computation of VaR for China securities markets and the Forex markets to demonstrate the proposed method. In Section 4, some discussions are given.

2. The computational framework for value-at-risk

In order to compute VaR for capital markets on the condition of non-random walks, we use Gaussian mixture to measure the risk of financial assets in capital markets. It is well known that Gaussian mixture can be used not only to fit returns of financial assets but also to capture non-normality of financial assets movements [11,12]. So, we pay attention to computational framework of VaR for capital markets with Gaussian mixture.

Let $S_t \in \Re$ be the financial asset prices series (stock, index, or exchange rate) and $\mathbb{S} = \{S_{-1}, S_0, S_1, \ldots, S_{n-1}\}$. Define returns of financial assets time series as

$$r_t = \log(S_t/S_{t-1}) \quad \forall S_t \in \mathbb{S} \ \ \forall t \in \{0, \ldots, n-1\} \tag{1}$$

and let $\mathbb{R} = \{x_0, x_1, \ldots, x_{n-1}\}$ and $S_{-1} = S_0$.

Suppose that a component distribution $f_k(r|\mu_k, \sigma_k^2)$ is Gaussian, i.e. $N(\mu_k, \sigma_k^2)$, named this component as ω_k. Let K be the number of components,