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ELSEVIER Applied Mathematics and Computation 168 (2005) 1251–1257
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Error estimation problem of nonlinear integral equations in the space $L_p (p \geqslant 1)$

M. El-kady [a,*], A. Abd El-Shafy [b]

[a] *Faculty of Science, Department of Mathematics, South Valley University, Aswan, Egypt*
[b] *Faculty of Education, Department of Mathematics, El-Arish-Suez Canal University (El-Arish), Egypt*

Abstract

The main purpose of this paper is to intoduce and study approximate solution method for nonlinear Volterra–Hammerstein integral equations with an arbitrary smooth (nonlinear) kernel function. At the end of this paper, numerical example is treated.

Keywords: Approximate solutions; Nonlinear integral equations; Optimal control problems

1. Introduction

Today, the theory of integral equations are one of the major are of applied mathematics. The theory of Hammerstein integral equations is a direct generalization of theory of ordinary differential equations. The degenerate kernel

* Corresponding author.

0096-3003/$ - see front matter © 2004 Published by Elsevier Inc.
doi:10.1016/j.amc.2004.10.017

method has been developed to find best approximate solutions. This method is based on the replacement the given equation by Hammerstein type with an arbitrary smooth degenerate kernel and take the solution of it as an approximate solution to the given equation. By transforming the new equation into a system of nonlinear algebraic equations, and we suggest, for computational purpose, a some what different view of the error estimation problem.

Let us consider the following nonlinear integral equation of the form:

$$y(x) = g(x) + \int_0^x V(x,t,y(t))\,dt + \int_0^{2\pi} L(x,t,y(t))\,dt, \tag{1.1}$$

where V, L and g are 2π-periodic functions.

The approximate solutions by using the linear polynomial operator methods was introduced in [1,4]. The classical method of the degenerate kernel is applied to numerical solutions in [5]. The question of existence and uniqueness of the solution to nonlinear integral equations be given in [6]. The simplicity of finding a solution of Hammeerstein integral equations with degenerate kernel. So, we can be written Eq. (1.1) of the form:

$$y(x) = g(x) + \int_0^{2\pi} H(x,t,y(t))\,dt, \tag{1.2}$$

where

$$H(x,t,y(t)) = K(x,t,y(t) + L(x,t,y(t))), \tag{1.3}$$

$$K(x,t,y(t)) = e(x,t,y(t))V(x,t,y(t)), \tag{1.4}$$

$$e(x,t,y(t)) = \left\{\begin{matrix} 1 & t \leqslant x \\ 0 & t > x \end{matrix}\right\}.$$

The function $H(x,t,y(t))$ satisfies uniformly a Lipschitz condition:

$$|H(x,t,y_1(t)) - H(x,t,y_2(t))| \leqslant N(x,t)|y_1(t) - y_2(t)|.$$

2. Error estimation

Lemma 1 [4]. *Let the kernel $H(x,t,y(t))$ be a 2π-periodic function, if*

$$H(x,t,y(t)) \in L_p[Q]; \quad Q = \{(x,t)\colon 0 \leqslant x,t \leqslant 2\pi\} \quad \textit{and}$$

$$y(x) \in L_p; \quad q = \frac{p}{p-1}.$$

Then

$$U_n\left[\int_0^{2\pi} H(.,t,y(t))\,dt;x\right] = \int_0^{2\pi} U_n[H(.,t,y(t))\,dt;x]\,dt.$$