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A reliable treatment of the physical structure for the nonlinear equation $K(m,n)$

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Abstract

In this work, the nonlinear equation $K(m,n)$ is studied for all possible values of m and n. We show that this equation may exhibit compactons, solitons or periodic solutions. The analysis reveals the change of the physical structure of the solutions as a result of the change of m and n.

Keywords: Compactons; Solitons; Nonlinear dispersion; Periodic solutions; Sine–cosine method

1. Introduction

The nonlinear term uu_x in the standard KdV equation

$$u_t + \alpha uu_x + u_{xxx} = 0, \tag{1}$$

causes the steepening of wave form. However, the dispersion effect term u_{xxx} in Eq. (1) makes the wave form spread [1,2]. The solitons exist as a result of the balance between the weak nonlinearity and dispersion. Wadati [1,2] and others have defined solitons as a nonlinear wave that has the following properties:

(1) A localized wave propagates without change of its properties (shape, velocity, etc.).

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(2) Localized waves are stable against mutual collisions and retain their identities. This means that the integrable nonlinear KdV Eq. (1) with linear dispersion admits solitons that are waves with infinite support or exponential wings.

However, some novel features may be observed when the wave dispersion is purely nonlinear. The nonintegrable nonlinear dispersive $K(n,n)$ equation

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, \quad n > 1 \tag{2}$$

with nonlinear convection term $(u^n)_x$ and genuinely nonlinear dispersive term $(u^n)_{xxx}$, has been examined in [3–19]. It was formally derived in these works that the delicate interaction between nonlinear convection with genuine nonlinear dispersion generates solitary waves with exact compact support that are termed *compactons* which vanish outside a finite core region. Unlike soliton that narrows as the amplitude increases, the compacton's width is independent of the amplitude. Two important features of compactons structures are observed:

(1) The compacton is a soliton characterized by the absence of exponential wings.
(2) The width of the compacton is independent of the amplitude.

In modern physics, a suffix-on is used to indicate the particle property [1], for example phonon, photon, and soliton. For this reason, the solitary wave with compact support is called *compacton* to indicate that it has the property of a particle.

Several significant studies have been developed in [3–19] to emphasize the fact that purely nonlinear dispersion can cause a deep qualitative change in the genuinely nonlinear phenomenon. It was shown that classical solitons are analytic solutions, whereas compactons are nonanalytic solutions as investigated in [3–7]. The points of nonanalyticity at the edge of the compacton correspond to points of genuine nonlinearity for the differential equation.

The stability analysis has shown that compacton solutions are stable, where the stability condition is satisfied for arbitrary values of the nonlinearity parameter. The stability of the compactons solutions was investigated by means of both linear stability and by Lyapunov stability criteria as well.

Three main methods, namely, the pseudo spectral method, the tri-Hamiltonian operators, and Adomian decomposition method [20–22], have been employed as appropriate schemes to handle the $K(n,n)$ equation analytically. However, a numerical approach based on the finite differences method was employed by Ismail and Taha [7] to address the instabilities on the discontinuous fronts. Other methods have been used recently.