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Optimal control of volterra equations with impulses

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Abstract

We consider an optimal control problem for a system governed by a Volterra integral equation with impulsive terms. The impulses act on both the state and the control; the control consists of switchings at discrete times. The cost functional includes both, an integrated cost rate (continuous part) and switching costs at the discrete impulse times (discrete part). We prove necessary optimality conditions of a form analogous to a discrete maximum principle. For the particular case of a system governed by impulsive ordinary differential equations, we obtain an impulsive maximum principle as a special case of the necessary optimality conditions for impulsive Volterra equations.

Keywords: Volterra equations with impulses; Optimal control; Maximum principle; Impulsive differential equations

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1. Introduction

In this paper, we obtain necessary conditions for optimality for Volterra integral equations with impulsive terms and piecewise constant controls. The cost functional that is to be minimized includes an integral term for the rate of cost per unit time, as well as discrete switching costs at the times of changing the values of the control. For simplicity, we consider only discrete controls and fixed switching times. The combination of continuous and discrete controls, and the case of variable switching times, are objects of ongoing work and will be reported elsewhere.

The novelty of the problem we consider here lies in the inclusion of impulsive terms in the Volterra integral equation. Without impulsive terms in the state dynamics, the case of piecewise constant controls has been treated in [12–14]. We note here that the general questions of optimality conditions for optimal control problems governed by Volterra integral equations have been studied in, among other references, [1,2,5,11–15].

It is well known that Volterra integral equations can be used to model many classes of phenomena, for example population dynamics, continuum mechanics of materials with memory, economic problems, the spread of epidemics, non-local problems of diffusion and heat conduction, etc. Some of these applications may be found in the classical references [6,9], and other examples can be found in more specialized works. The corresponding control problems for such systems lead to optimal control problems for Volterra integral equations. An explicit example of applying the methods of optimal control of Volterra integral equations to economics is the paper [8].

All kinds of control systems can be subjected to impulsive conditions. Impulsive control systems may arise from state models that are intrinsically impulsive, i.e. the physical model without a control function (the uncontrolled model) still involves impulsive terms, as it happens, for example, in systems of variable structure when a system involves transitions through different operating regimes. Impulsive control problems can also arise because the action of the control may involve impulses applied to the state of the system, as, for example, investment decisions in economics, or the injection of a medical drug into a patient in mathematical models in pharmacology. Therefore, it is of interest to study the optimal control of impulsive Volterra integral equations.

The mathematical analysis of problems with discrete controls is useful not only when the original formulation of the state dynamics of a controlled system involves discrete controls, but also when one tries to numerically solve an optimal control problem that was originally formulated as a problem with continuous controls. The numerical solution of optimal control problems often involves a discretization of the control function (see, e.g., [16] for discrete approximations to ODE control problems), and in that case one has to solve the corresponding optimal control problem with a discrete-valued control function.