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Analysis of periodically time-varying discrete-time systems in spectral domain

Gülay Tohumoğlu

Department of Electrical and Electronics Engineering, University of Gaziantep, 27310 Sehitkamil, Gaziantep, Turkey

Abstract

It is well known the concept of the steady-state response in the theory of linear time invariant systems. In this paper, the method called spectral analysis is newly adopted for periodically time varying (PTV) discrete time systems from the theory of the spectral analysis of linear PTV continuous time systems. Shortly, it describes the steady-state analysis of PTV discrete time systems. It leads to a simple algebraic relations in the frequency domain that minimizes calculations in the time domain. In order to show the generality and usefulness of this method, an illustrative examples are chosen. The comparison of the results with the exact solution concludes that the systems having smooth variation-less number of harmonics with respect to strongly varying systems are rapidly convergent with the high accuracy. The error analysis has being done by computing the root-mean-square error.

Keywords: Periodic systems; Spectral analysis; Spectral domain; Time-varying systems; Discrete-time systems

1. Introduction

The analysis of systems with the time-varying elements is more complicated than the analysis of systems with constant elements called time invariant. The solution of system of equations like differential equations in the time-domain, however, leads to numerical efforts considerably, frequency domain analysis

E-mail address: g_tohumoglu@gantep.edu.tr (G. Tohumoğlu).

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are well established for the analysis of time-invariant systems. All these methods were almost applied and modified for the analysis of time-varying systems [1–3]. Although linear PTV systems form a subclass of time-varying systems, still it has interest in practice. Since the complete solution can be computed for only a very narrow class of linear PTV systems, general methods such as spectral analysis [4,5] and piecewise-constant approximation method are of interest in many cases [6]. The latter one is a time domain and the spectral analysis method uses the advantages of frequency domain either experimentally and/or theoretically by the developed algebraic relations.

The advent of discrete-time signal processing systems such as digital filters, switched capacitor filters, multirate filters has always interest in the theory of linear (periodically) time-varying systems. In this study, the spectral theory is unified and newly adapted to the linear PTV discrete time systems by defining simple algebraic relation matrices. This is so because the existing concepts are either use recursive relation [3] or equivalent time-invariant block representation of Mehr and Chen [7]. Here, it is originally and mainly emphasized that the spectral analysis method is very suitable to adopt for the linear PTV discrete time systems by using Discrete Fourier series. Thus, the different forms of system describing equations can be transformed into the spectral domain by forming matrix-vector algebraic relations.

The following part of the paper briefly describes a general introduction of the theory of linear PTV continuous time systems. The particular form of the system equations as differential equation, state-space representation, transfer function are given and the corresponding input–output spectral relation is considered. Basically, this section serves both for giving comprehensive survey of system describing equations as well as for summarizing the background and notations needed in the remaining part of the paper.

In Section 3, the general aspects of discrete time systems are given. In Section 4, the extension of the spectral theory is newly applied for the linear PTV discrete-time system. The discrete-time system equations in different forms are transferred to the spectral domain in terms of the specially defined modem, delay etc. matrices and related spectral vectors together.

Finally, illustrative examples are chosen. All derivation steps in order to transfer them into the spectral domain are shown. The quantitative analysis of the results by means of root mean square error computation is done and then conclusions are emphasized in summary.

2. Spectral analysis method

This section is intended to provide a thorough survey covering the spectral analysis formulations. There are many issues encountered on this subject that can be considered as novel contributions. However, this work adopts the