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Convergence of a conservative difference scheme for a class of Klein–Gordon–Schrödinger equations in one space dimension

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Abstract

A conservative difference scheme is presented for the initial-boundary value problem of a class of Klein–Gordon–Schrödinger equations. The scheme can be implicit or implicit-explicit, depending on the choice of a parameter. On the basis of the priori estimates and an inequality about norms, convergence of the difference solution is proved with order $O(h^2 + \tau^2)$ in the energy norm.

Keywords: Klein–Gordon–Schrödinger equations; Difference scheme; Implicit-explicit scheme; Difference solution; Convergence

1. Introduction

Klein–Gordon–Schrödinger equations [1]

$$\mathrm{i}\frac{\partial \varphi}{\partial t} + \frac{1}{2}\frac{\partial^2 \varphi}{\partial x^2} + u\varphi = 0, \tag{1.1}$$

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u - |\varphi|^2 = 0, \tag{1.2}$$

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are a classical model described the interaction between conservative complex neutron field and neutral meson Yukawa in quantum field theory. Here $\varphi(x,t)$ is a complex function, $u(x,t)$ is a real function and $\mathrm{i}^2=-1$. Global strong solutions in [2,3] and stability of stationary states in [4] is proved for the Klein–Gordon–Schrödinger equations. In [1,5], asymptotic behavior of the solutions is reported. The solitary wave solutions of the Klein–Gordon–Schrödinger equations are obtained in [6].

But, numerical studies for the Klein–Gordon–Schrödinger equations are few. We constructed a implicit scheme in [7]. However, many finite difference schemes have been presented [8–12] for the Zakharov equations:

$$\mathrm{i}\frac{\partial\varphi}{\partial t}+\frac{\partial^2\varphi}{\partial x^2}-N\varphi=0,$$
$$\frac{1}{\lambda^2}\frac{\partial^2 N}{\partial t^2}-\frac{\partial^2 N}{\partial x^2}-\frac{\partial^2|\varphi|^2}{\partial x^2}=0.$$

The two couple equations are very similar. Thus, we expect to have similar numerical results.

In this paper, we consider the numerical solution of the Klein–Gordon–Schrödinger equations with the following initial value and boundary value conditions.

$$\varphi|_{t=0}=\varphi_0(x),\quad u|_{t=0}=u_0(x),\quad u_t|_{t=0}=u_1(x), \tag{1.3}$$

$$\varphi|_{x=X_L}=\varphi|_{x=X_R}=0,\quad u|_{x=X_L}=u|_{x=X_R}=0, \tag{1.4}$$

where $\varphi_0(x)$, $u_0(x)$ and $u_1(x)$ are known smooth functions.

The initial-boundary-value problem (1.1)–(1.4) possesses tow conservative quantities.

$$\|\varphi\|^2=E_0, \tag{1.5}$$

$$E(t)=\frac{1}{2}\left(\|u\|^2+\|u_t\|^2+\|u_x\|^2+\|\varphi_x\|^2\right)-\left(|\varphi|^2,u\right)=E_1, \tag{1.6}$$

where E_0 and E_1 are constants which depend only on initial value.

We propose a new conservative difference scheme which involves a parameter θ, $0\leqslant\theta\leqslant\frac{1}{2}$. When $\theta=\frac{1}{2}$, the new scheme is identical to the scheme in [7]. For $\theta=0$, the new scheme is implicit-explicit, implicit in φ, but explicit in u. The truncation error of the new scheme is $\mathrm{O}(h^2+\tau^2)$. A priori estimates for the difference solution will be made, and a useful inequality about norms will be obtained. Convergence of the difference solution in energy norm will be proved based on the inequality. The result which the difference scheme is convergent for all $0\leqslant\theta\leqslant\frac{1}{2}$ is significant because, although the convergence of the implicit scheme ($\theta=\frac{1}{2}$) has been known [7], the convergence of the more general case ($\theta\neq\frac{1}{2}$) is more difficult for prove.