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On computing of arbitrary positive integer powers for one type of odd order symmetric circulant matrices–I

Jonas Rimas

Department of Applied Mathematics, Faculty of Fundamental Sciences, Kaunas University of Technology, Studentu 50, Kaunas 51368, Lithuania

Abstract

In this paper we derive the general expression of the lth power ($l \in N$) for one type of symmetric circulant matrix of order $n = 2p + 1$ ($p \in N$).

Keywords: Eigenvalues; Eigenvectors; Circulant matrices; Jordan's form; Chebyshev polynomials

1. Introduction

Solving some difference and differential equations and delay differential equations we meet the necessity to compute the arbitrary positive integer powers of square matrix [1–3]. In this paper we derive the general expression of the lth power ($l \in N$) for one type of symmetric circulant matrices [4].

E-mail addresses: jonas.rimas@ktu.lt, jonas.rimas@fmf.ktu.lt

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2. Derivation of general expression

Consider the nth order ($n = 2p + 1$, $p \in N$) symmetric circulant matrix B of the following type:

$$B = \begin{pmatrix} 0 & 1 & & & & 1 \\ 1 & 0 & 1 & & & \\ & 1 & 0 & 1 & & \\ & & & \ddots & & \\ & & & 1 & 0 & 1 \\ 1 & & & & 1 & 0 \end{pmatrix}. \tag{1}$$

The lth power ($l \in N$) of this matrix we will find using expression $B^l = TJ^lT^{-1}$ [5], where J is the Jordan's form of B, T is the transforming matrix. Matrices J and T can be found, provided eigenvalues and eigenvectors of the matrix B are known. The eigenvalues of B are defined by the characteristic equation

$$|B - \lambda E| = 0. \tag{2}$$

Let us denote

$$D_n(\alpha) = \begin{vmatrix} \alpha & 1 & & & & 1 \\ 1 & \alpha & 1 & & & \\ & 1 & \alpha & 1 & & \\ & & & \ddots & & \\ & & & 1 & \alpha & 1 \\ 1 & & & & 1 & \alpha \end{vmatrix}, \qquad \Delta_n(\alpha) = \begin{vmatrix} \alpha & 1 & & & & \\ 1 & \alpha & 1 & & 0 & \\ & 1 & \alpha & 1 & & \\ & & & \ddots & & \\ & 0 & & 1 & \alpha & 1 \\ & & & & 1 & \alpha \end{vmatrix}. \tag{3}$$

here $\alpha \in R$. Then

$$|B - \lambda E| = D_n(-\lambda). \tag{4}$$

From (3) follows

$$D_n = \alpha\Delta_{n-1} - 2\Delta_{n-2} - 2(-1)^n \tag{5}$$

and

$$\Delta_n = \alpha\Delta_{n-1} - \Delta_{n-2} \ (\Delta_2 = \alpha^2 - 1, \Delta_1 = \alpha, \Delta_0 = 1); \tag{6}$$

here $D_n = D_n(\alpha)$, $\Delta_n = \Delta_n(\alpha)$. Solving difference equation (6) we obtain

$$\Delta_n(\alpha) = U_n\left(\frac{\alpha}{2}\right), \quad D_n(\alpha) = U_n\left(\frac{\alpha}{2}\right) - U_{n-2}\left(\frac{\alpha}{2}\right) - 2(-1)^n;$$