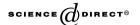


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# On computing of arbitrary positive integer powers for one type of odd order symmetric circulant matrices—I

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#### Abstract

In this paper we derive the general expression of the lth power ( $l \in N$ ) for one type of symmetric circulant matrix of order n = 2p + 1 ( $p \in N$ ). © 2004 Published by Elsevier Inc.

Keywords: Eigenvalues; Eigenvectors; Circulant matrices; Jordan's form; Chebyshev polynomials

#### 1. Introduction

Solving some difference and differential equations and delay differential equations we meet the necessity to compute the arbitrary positive integer powers of square matrix [1–3]. In this paper we derive the general expression of the lth power ( $l \in N$ ) for one type of symmetric circulant matrices [4].

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### 2. Derivation of general expression

Consider the *n*th order  $(n = 2p + 1, p \in N)$  symmetric circulant matrix *B* of the following type:

$$B = \begin{pmatrix} 0 & 1 & & & & 1 \\ 1 & 0 & 1 & & & & \\ & 1 & 0 & 1 & & & \\ & & & \ddots & & & \\ & & & 1 & 0 & 1 \\ 1 & & & & 1 & 0 \end{pmatrix}. \tag{1}$$

The *l*th power  $(l \in N)$  of this matrix we will find using expression  $B^l = TJ^lT^{-1}$  [5], where *J* is the Jordan's form of *B*, *T* is the transforming matrix. Matrices *J* and *T* can be found, provided eigenvalues and eigenvectors of the matrix *B* are known. The eigenvalues of *B* are defined by the characteristic equation

$$|B - \lambda E| = 0. (2)$$

Let us denote

$$D_{n}(\alpha) = \begin{vmatrix} \alpha & 1 & & & & 1 \\ 1 & \alpha & 1 & & & & \\ & 1 & \alpha & 1 & & & \\ & & & \ddots & & \\ & & & 1 & \alpha & 1 \\ 1 & & & & 1 & \alpha \end{vmatrix}, \qquad \Delta_{n}(\alpha) = \begin{vmatrix} \alpha & 1 & & & & \\ 1 & \alpha & 1 & & & 0 \\ & 1 & \alpha & 1 & & & \\ & & & \ddots & & & \\ & 0 & & 1 & \alpha & 1 \\ & & & & 1 & \alpha \end{vmatrix}.$$

$$(3)$$

here  $\alpha \in R$ . Then

$$|B - \lambda E| = D_n(-\lambda). \tag{4}$$

From (3) follows

$$D_n = \alpha \Delta_{n-1} - 2\Delta_{n-2} - 2(-1)^n \tag{5}$$

and

$$\Delta_n = \alpha \Delta_{n-1} - \Delta_{n-2} \ (\Delta_2 = \alpha^2 - 1, \Delta_1 = \alpha, \Delta_0 = 1);$$
(6)

here  $D_n = D_n(\alpha)$ ,  $\Delta_n = \Delta_n(\alpha)$ . Solving difference equation (6) we obtain

$$\Delta_n(\alpha) = U_n\left(\frac{\alpha}{2}\right), \quad D_n(\alpha) = U_n\left(\frac{\alpha}{2}\right) - U_{n-2}\left(\frac{\alpha}{2}\right) - 2(-1)^n;$$

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