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Rotating flow of a third grade fluid by homotopy analysis method

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Abstract

The steady flow of a rotating third grade fluid past a porous plate has been analyzed. The resulting nonlinear boundary value problem has been solved using homotopy analysis method. Explicit expression for the velocity field has been obtained. The variations of velocity with respect to rotation, suction, blowing and non-Newtonian parameters are shown and discussed.

Keywords: Rotating flow; Third grade fluid; Homotopy analysis method

1. Introduction

The analysis of the effects of rotation in fluid flows has been an interesting area because of its geophysical and technological importance. The involved equations are nonlinear and thus to understand specific aspects of the fluid flow simplified models have been taken into account. In this work, the steady-state flow of an incompressible fluid past a porous plate is considered. The fluid is

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third grade and the whole system is in a rotating frame. Both (analytical and graphical) solutions of the governing nonlinear differential equation is given. Analytic solution of the problem is given by a newly developed method known as homotopy analysis method by Liao [1]. This method has already been successfully applied by various workers [2–8]. Briefly, the homotopy analysis method has the following advantages:

- It is independent of the choice of any large/small parameters in the nonlinear problem.
- It is helpful to control the convergence of approximation series in a convenient way and also for the adjustment of convergence regions where necessary.
- It can be employed to efficiently approximate a nonlinear problem by choosing different sets of base functions.

The layout of the paper is:

In Section 2, the problem is formulated. The solution of the problem is given in Section 3. Section 4 deals with the discussion of several graphs and in Section 5, concluding remarks are presented.

2. Mathematical formulation

We consider a Cartesian coordinate system rotating uniformly with an angular velocity Ω about the z-axis, taken positive in the vertically upward direction, with the plate coinciding with the plane $z = 0$. The fluid past a porous plate is third grade and incompressible. All material parameters of the fluid are assumed constant. In rotating frame, the momentum equation is

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} + 2\Omega\times\mathbf{V} + \Omega\times(\Omega\times\mathbf{r})\right] = \mathrm{div}\mathbf{T}. \tag{1}$$

In above equation ρ is the density of the fluid, $\mathbf{r}$ is the radial coordinate and $\mathbf{V}$ is the velocity. The Cauchy stress tensor $\mathbf{T}$ for third grade fluid is [9]

$$\mathbf{T} = -p_1\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(\mathrm{tr}\mathbf{A}_1^2)\mathbf{A}_2 \tag{2}$$

in which p_1 is the pressure, $\mathbf{I}$ is the identity tensor, μ is the dynamic viscosity, α_i $(i = 1,2)$, β_i $(i = 1,2,3)$ are the material constants and the Rivlin–Erickson tensors are defined by

$$\mathbf{A}_1 = (\mathrm{grad}\mathbf{V}) + (\mathrm{grad}\mathbf{V})^{\mathrm{T}}, \tag{3}$$

$$\mathbf{A}_{n+1} = \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{A}_n + (\mathrm{grad}\mathbf{V})^{\mathrm{T}}\mathbf{A}_n + \mathbf{A}_n(\mathrm{grad}\mathbf{V}), \quad n > 1. \tag{4}$$