Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 161 (2005) 395–401
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Periodicity and attractivity of a nonlinear higher order difference equation

Guang Zhang *, Lijie Zhang

Department of Mathematics, Qingdao Institute of Architecture and Engineering, Qingdao, Shandong 266033, PR China

Abstract

In this paper, a nonlinear higher order difference equation is considered. The existence and nonexistence of two-period positive solution and the attractivity are investigated.

Keywords: Difference equation; Higher order; Periodicity; Attractivity

1. Introduction

In this paper, we will consider the difference equation of the form

$$x_{n+1} = \frac{a - bx_{n-k}}{A + x_{n-l}}, \quad n = 0, 1, \ldots, \tag{1}$$

where $a \geqslant 0$, $A, b > 0$ are real numbers, k and l are integer numbers. The periodicity and attractivity of (1) are discussed.

Clearly, the following equations are particular cases of (1):

$$x_{n+1} = \frac{a - bx_{n-1}}{A + x_n}, \quad n = 0, 1, \ldots, \tag{2}$$

* Corresponding author.
E-mail addresses: dtgzhang@yahoo.com.cn, dtguangzhang@yahoo.com.cn (G. Zhang).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2003.12.034

$$x_{n+1} = \frac{a - bx_n}{A + x_{n-l}}, \quad n = 0, 1, \ldots, \tag{3}$$

$$x_{n+1} = \frac{a - bx_n}{A + x_{n-1}}, \quad n = 0, 1, \ldots, \tag{4}$$

and

$$x_{n+1} = \frac{a - bx_{n-k}}{A + x_n}, \quad n = 0, 1, \ldots \tag{5}$$

Eqs. (2)–(5) have been studied in [1–5].

2. Periodicity

In this section, we will discuss the existence and nonexistence of two-period positive solutions of Eq. (1).

Theorem 1. *Assume that* $A > b > 0$. *Then Eq.* (1) *has no two-period positive solution.*

Proof. Assume for the sake of contradiction that there exist distinctive positive real numbers ϕ and ψ, such that

$$\ldots, \phi, \psi, \phi, \psi, \ldots,$$

be a two-period solution of (1). There are four cases to be considered.

Case (a) k and l are odd.

In this case $x_{n+1} = x_{n-k} = x_{n-l}$, ϕ and ψ satisfy the system

$$\phi(A + \phi) = a - b\phi \quad \text{and} \quad \psi(A + \psi) = a - b\psi,$$

which has unique nonnegative solution

$$\phi = \psi = \frac{-(A + b) + \sqrt{(A + b)^2 + 4a}}{2},$$

this is a contradiction.

Case (b) k and l are even.

In this case $x_{n-k} = x_{n-l}$, ϕ and ψ satisfy the system

$$\phi(A + \psi) = a - b\psi \quad \text{and} \quad \psi(A + \phi) = a - b\phi,$$

which implies

$$A(\phi - \psi) = b(\phi - \psi),$$

so $\phi = \psi$ which contradicts the hypothesis $\phi \neq \psi$.