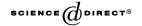
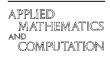


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Periodicity and attractivity of a nonlinear higher order difference equation

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Abstract

In this paper, a nonlinear higher order difference equation is considered. The existence and nonexistence of two-period positive solution and the attractivity are investigated.

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1. Introduction

In this paper, we will consider the difference equation of the form

$$x_{n+1} = \frac{a - bx_{n-k}}{A + x_{n-l}}, \quad n = 0, 1, \dots,$$
 (1)

where $a \ge 0$, A, b > 0 are real numbers, k and l are integer numbers. The periodicity and attractivity of (1) are discussed.

Clearly, the following equations are particular cases of (1):

$$x_{n+1} = \frac{a - bx_{n-1}}{A + x_n}, \quad n = 0, 1, \dots,$$
 (2)

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$$x_{n+1} = \frac{a - bx_n}{A + x_{n-1}}, \quad n = 0, 1, \dots,$$
 (3)

$$x_{n+1} = \frac{a - bx_n}{A + x_{n-1}}, \quad n = 0, 1, \dots,$$
 (4)

and

$$x_{n+1} = \frac{a - bx_{n-k}}{A + x_n}, \quad n = 0, 1, \dots$$
 (5)

Eqs. (2)–(5) have been studied in [1–5].

2. Periodicity

In this section, we will discuss the existence and nonexistence of two-period positive solutions of Eq. (1).

Theorem 1. Assume that A > b > 0. Then Eq. (1) has no two-period positive solution.

Proof. Assume for the sake of contradiction that there exist distinctive positive real numbers ϕ and ψ , such that

$$\ldots, \phi, \psi, \phi, \psi, \ldots,$$

be a two-period solution of (1). There are four cases to be considered.

Case (a) k and l are odd.

In this case $x_{n+1} = x_{n-k} = x_{n-l}$, ϕ and ψ satisfy the system

$$\phi(A + \phi) = a - b\phi$$
 and $\psi(A + \psi) = a - b\psi$,

which has unique nonnegative solution

$$\phi = \psi = \frac{-(A+b) + \sqrt{(A+b)^2 + 4a}}{2},$$

this is a contradiction.

Case (b) k and l are even.

In this case $x_{n-k} = x_{n-l}$, ϕ and ψ satisfy the system

$$\phi(A + \psi) = a - b\psi$$
 and $\psi(A + \phi) = a - b\phi$,

which implies

$$A(\phi - \psi) = b(\phi - \psi),$$

so $\phi = \psi$ which contradicts the hypothesis $\phi \neq \psi$.

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