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The venerable 1/7th power law turbulent velocity profile: a classical nonlinear boundary value problem solution and its relationship to stochastic processes

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Abstract

The 1/7th power law turbulent velocity profile, originally estimated from pipe flow data, provided and continues to provide a simple but effective relationship for turbulent mean velocity profiles in moderate favorable pressure gradient regime flows. Here we show that the power law profile is not only a viable empirical relationship but the analytical solution of a nonlinear boundary value problem based on a large Reynolds number asymptotic closures. By extending the length scale closure to include related elementary models we obtain other mean velocity profiles solutions that are in good agreement with the more well-known power-law solution. The interest in these other closure models is that they exhibit a direct connection to classical, normally distributed stochastic process behavior. This relationship is further explored by considering a discrete single step difference equation governing the "random walk behavior" of fluid particle in a wall bounded shear flow. Thus, from these simple analytical solutions, we can relate the success of the empirical 1/7th power law model to a more fundamental understanding of turbulent flow, turbulence modeling closures and their connection to stochastic processes.

Keywords: 1/7th power law profile; Fully developed turbulent flow; Random processes

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Nomenclature

a	streamwise dependent separation variable
C, c	constant, integration constants
D	diffusion constant
f	cross-stream dependent separation variable
M	Mach number
p	Reynolds averaged pressure
R	Pipe radius
Re	Reynolds number
t	time
u	Reynolds averaged streamwise velocity
U	free stream velocity
x	streamwise spatial coordinate
y	cross-stream spatial coordinate
Greeks	
α, β	proportionality constants
δ	boundary layer thickness
Δ	finite change
ξ	fluctuation weighting function
γ	ratio of specific heats
λ	separation constant
ν	kinematic viscosity constant
ϕ	transported fluid property
Subscript/superscript	
eff	effective value
$'$	stochastic/turbulent fluctuation quantity

1. Introduction

The use of empirically based power law functions to describe mean turbulent for both fully developed pipe flow starting with Nikuradse, as discussed in [1] and, with extensions for flat plate flows has been an approximate, but successful modeling strategy for many years [1,2]. Even with current computational and modeling assets capable of performing large scale, engineering relevant turbulent flow simulations, description of the mean flow profile using simple relationships provides valuable insight. Indeed the continued viability of the power profile concept is demonstrated by a sampling of current papers that employ the power law profile as a part of their research effort, e.g. [3–5]. A theoretical basis for high-Reynolds number power-law relationships, as