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Nonlinear dispersive special type of the Zakharov–Kuznetsov equation ZK(n, n) with compact and noncompact structures

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Abstract

In this paper we study nonlinear dispersive special type of the Zakharov–Kuznetsov equation with positive and negative exponents. The approach depends mainly on the sine–cosine algorithm. Compactons, solitary patterns, solitons, and periodic solutions are formally derived.

Keywords: Compactons; Solitons; Periodic solutions; Zakharov–Kuznetsov equation; KP equation; KdV equation; Sine–cosine method

1. Introduction

The KdV equation is a model that governs the one-dimensional propagation of small-amplitude, weakly dispersive waves [1,2]. The nonlinear term uu_x in the KdV equation

$$u_t + auu_x + u_{xxx} = 0, \tag{1}$$

causes the steepening of wave form, whereas the dispersion effect term u_{xxx} in the same equation makes the wave form spread. The balance between this weak nonlinear steepening and dispersion gives rise to solitons. The KdV equation is therefore incapable of shock waves [3]. The KdV equation describes the evolution of the weakly nonlinear steepening and the weakly dispersive wave that

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appears in many applications such as surface waves in shallow water, plasma physics, acoustic waves, heat pulses in anharmonic crystals, ion-acoustic wave, stratified internal waves, and magneto-sonic waves in a magnetized plasma. The KdV equation plays an important role in the development of the soliton theory, where nonlinearity and dispersion dominate, while dissipation effects are small enough to be neglected in the lowest order approximation [4,5].

Soliton is a localized wave that has an infinite support or a localized wave with exponential wings. Wadati [6–8] defined soliton as a nonlinear wave that has the following properties:

(1) A localized wave propagates without change of its properties (shape, velocity, etc.).
(2) Localized waves are stable against mutual collisions and retain their identities. This means that soliton has the property of a particle.

However, the KdV equation is considered a spatially one-dimensional model. An extensive research work has been done in developing higher dimensional models, particularly those in the $(2+1)$, two spatial and one time, dimensions [9]. The best known two-dimensional generalizations of the KdV equations are the Kadomtsev–Petviashivilli (KP) equation, and the Zakharov–Kuznetsov (ZK) equation. The KP equation is given by

$$\{u_t + auu_x + u_{xxx}\}_x + u_{yy} = 0. \tag{2}$$

The KP equation has been studied in many papers to the soliton self-focusing, because it commonly appears in different physical applications, and it is integrable by means of the inverse scattering transform method [4,10]. The integrable KP equation is a higher dimensional model that characterizes, small-amplitude, weakly dispersive waves on a fluid sheet. In other words, the KP equation is a weakly 2D generalization in the sense that it accounts for slowly varying transverse perturbations of unidirectional KdV solitons moving along the x-direction [11].

The Zakharov–Kuznetsov (ZK) equation is another alternative version of a nonlinear model describing two-dimensional modulations of a KdV soliton [10,12]. If a magnetic field is directed along the x-axis, the Zakharov–Kuznetsov equation in renormalized variables [10] takes the form

$$u_t + auu_x + (\nabla^2 u)_x = 0, \tag{3}$$

where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the isotropic Laplacian. This means that the ZK equation is given by

$$u_t + auu_x + (u_{xx} + u_{yy})_x = 0, \tag{4}$$