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Collocation solution for RLW equation with septic spline

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Abstract

A collocation solution using the septic splines as a shape function for the regularized long wave (RLW) equation is presented here. A linear stability analysis shows the scheme to be unconditionally stable. Test problems which, migration and interaction of solitary waves, are used to validate our scheme by calculate L_2-norm and L_∞-norm, and three invariants of motion are evaluates to determine the conservation properties of the algorithm. The numerical scheme is compared with other published methods and shown to be accurate and efficient

Keywords: RLW equation; Collocation methods; Splines; Solitary waves

1. Introduction

The regularized long wave equation (RLW) is an important non-linear wave equation. Solitary waves are wave packets or pulses which propagate in non-linear media. Due to dynamical balance between the non-linear and dispersive effects these waves retain a stable waveform. RLW equation is alternative description of non-linear dispersive waves to the more usual KDV equation [1].

Numerical solutions based on finite difference method [3], and Rung–Kutta method [4] and Galerkin's method [5] have been given. Alexander and Morris [5] constructed a global trial function mainly from cubic B-splines. Gardner

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and Gardner [6], using the Galerkin's method and cubic B-splines as element shape function to construct an implicit finite element solution. The least squares method using linear space–time finite elements used to solve the RLW equation [7], in the end Soliman and Raslan [8] solved the RLW equation by using collocation method using quadratic B-spline at the mid point.

In this paper, we study (RLW) equation by collocation method with septic splines. The element matrices are determined algebraically and the equations governing the problem are obtained by explicitly assembling together the element matrices to obtain the full global matrix equation. The time integration used to solve the resulting system of ordinary differential equations involve a Crank–Niclson scheme, a linear stability analysis of the numerical scheme shows it is an unconditionally stable.

The collocation method with septic splines is shown to represent accurately the migration of single solitary wave. The interaction of two solitary waves is then studied. Finally the evaluation of Maxwellian initial condition into stable solitary waves is investigated.

2. Governing equations and collocation solutions

The RLW equation take the form

$$V_t + V_x + \varepsilon V V_x - \mu V_{xxt} = 0, \tag{1}$$

where μ and ε are positive parameters and the subscripts x and t denote the differentiation with the boundary condition $V \to 0$ as $x \to \pm\infty$. Using the mapping $\varepsilon U = \varepsilon V + 1$ we can transform this equation to

$$U_t + \varepsilon U U_x - \mu U_{xxt} = 0. \tag{2}$$

The boundary conditions will be chosen from,

$$\begin{aligned} &U(a,t) = U(b,t) = \frac{1}{\varepsilon}, \quad U_x(a,t) = U_x(b,t) = 0, \\ &U_{xxx}(a,t) = U_{xxx}(b,t) = 0. \end{aligned} \tag{3}$$

Let us consider $\tau : a = x_0 < x_1 < \cdots < x_N = b.$ as a partition of $[a,b]$ by knots x_i and let $\Psi_i(x)$ be those septic splines with knots of the points of τ. Then $\{\psi_{-3}(x), \psi_{-2}(x), \ldots, \psi_{N+3}(x)\}$ form a basis for functions defined over $[a,b]$. The numerical solution $U_N(x,t)$ to $U(x,t)$ take the form

$$U_N(x,t) = \sum_{l=-3}^{N+3} \gamma_l(t)\psi_l(x), \tag{4}$$