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Convergence of nonstationary multisplitting methods using ILU factorizations

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Abstract

In this paper, we first study convergence of nonstationary multisplitting methods associated with a multisplitting which is obtained from the ILU factorizations for solving a linear system whose coefficient matrix is a large sparse H-matrix. We next study a parallel implementation of the *relaxed nonstationary two-stage multisplitting method* (called Algorithm 2 in this paper) using ILU factorizations as inner splittings and an application of Algorithm 2 to parallel preconditioner of Krylov subspace methods. Lastly, we provide parallel performance results of both Algorithm 2 using ILU factorizations as inner splittings and the BiCGSTAB with a parallel preconditioner which is derived from Algorithm 2 on the IBM p690 supercomputer.

MSC: 65F10; 65F15

Keywords: Nonstationary multisplitting method; ILU factorization; H-matrix; Multisplitting; Krylov subspace method

1. Introduction

In this paper, we consider parallel nonstationary multisplitting methods for solving a linear system of the form

$$Ax = b, \quad x, b \in \mathbb{R}^n, \tag{1}$$

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where $A \in \mathbb{R}^{n\times n}$ is a large sparse H-matrix. Multisplitting method was introduced by O'Leary and White [14] and was further studied by many authors [6,13,19,23]. The multisplitting method can be thought of as an extension and parallel generalization of the classical block Jacobi method [3].

A representation $A = M - N$ is called a *splitting* of A when M is nonsingular. A splitting $A = M - N$ is called *regular* if $M^{-1} \geqslant 0$ and $N \geqslant 0$, and it is called *weak regular* if $M^{-1} \geqslant 0$ and $M^{-1}N \geqslant 0$ [1]. A collection of triples (M_k, N_k, E_k), $k = 1, 2, \ldots, \ell$, is called a *multisplitting* of A if $A = M_k - N_k$ is a splitting of A for $k = 1, 2, \ldots, \ell$, and E_k's, called weighting matrices, are nonnegative diagonal matrices such that $\sum_{k=1}^{\ell} E_k = I$. The *relaxed nonstationary multisplitting method* associated with this multisplitting and a positive relaxation parameter ω for solving a linear system $Ax = b$ is as follows.

Algorithm 1. Relaxed nonstationary multisplitting method

Given an initial vector x_0
For $i = 1, 2, \ldots,$ until convergence
  For $k = 1$ to ℓ
    $y_{k,0} = x_{i-1}$
    For $j = 1$ to $s(k, i)$
      $M_k y_{k,j} = N_k y_{k,j-1} + b$
  $x_i = \omega \sum_{k=1}^{\ell} E_k y_{k,s(k,i)} + (1 - \omega)x_{i-1}.$

Notice that Algorithm 1 with $\omega = 1$ is called the *nonstationary multisplitting method*. Mas et al. [10] showed the convergence of Algorithm 1 under certain conditions when A is an H-matrix. When (M_k, N_k, E_k), $k = 1, 2, \ldots, \ell$, is a multisplitting of A and $M_k = B_k - C_k$ is a splitting of M_k for each k, the *relaxed nonstationary two-stage multisplitting method* with a positive relaxation parameter ω for solving a linear system $Ax = b$ is as follows.

Algorithm 2. Relaxed nonstationary two-stage multisplitting method

Given an initial vector x_0
For $i = 1, 2, \ldots,$ until convergence
  For $k = 1$ to ℓ
    $y_{k,0} = x_{i-1}$
    For $j = 1$ to $s(k, i)$
      $y_{k,j} = \omega B_k^{-1}(C_k y_{k,j-1} + N_k x_{i-1} + b) + (1 - \omega) y_{k,j-1}$
  $x_i = \sum_{k=1}^{\ell} E_k y_{k,s(k,i)}.$

In Algorithm 2, the splittings $A = M_k - N_k$ are called outer splittings and the splittings $M_k = B_k - C_k$ are called inner splittings. Bru et al. [3] showed the convergence of Algorithm 2 when A is a monotone matrix (i.e., $A^{-1} \geqslant 0$) or A is an H-matrix. If $\omega = 1$ in Algorithm 2, then Algorithm 2 reduces to the *nonstationary two-stage multisplitting method*. Notice that the loop k of Algorithms 1 and 2 can be executed completely in parallel by different processors. Also notice that the number of inner iterations $s(k, i)$ in Algorithms 1 and 2 depends on the iteration i and the splitting $A = M_k - N_k$. Throughout the paper, it is assumed that