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JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
ELSEVIER
Journal of Computational and Applied Mathematics 182 (2005) 447–471
www.elsevier.com/locate/cam

Sequential systems of linear equations method for general constrained optimization without strict complementarity☆

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Received 28 October 2004; received in revised form 21 December 2004

Abstract

In this paper, based on an $l_1 - l_\infty$ hybrid exact penalty function, we proposed an infeasible SSLE method for general constrained optimization. An automatic adjustment rule is incorporated in the algorithm for the choice of the penalty parameter, which ensures that the penalty parameter be updated only finitely many times. We also extend the Facchinei–Fischer–Kanzow active-set identification technique to general constrained optimization and a corresponding identification function is given. At each iteration, only two or three reduced linear equations with the same coefficients are solved to obtain the search direction. Under the linear independence condition, the sequence generated by the new algorithm globally converges to a KKT point. In particular, the convergence rate is proved to be one-step superlinear without assuming the strict complementarity and under a condition weaker than the strong second-order sufficiency condition. Some preliminary numerical results indicate that the new algorithm is quite promising.

MSC: 90C30; 65K10

Keywords: Sequential systems of linear equations method; Linear independence; Strict complementarity; Global convergence; Superlinear convergence

☆ This project is supported by the National Science Foundation of China (10171055).
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0377-0427/$ - see front matter
doi:10.1016/j.cam.2004.12.023

1. Introduction

In this paper, we consider the following equality and inequality constrained optimization problem:

$$\text{(P)}: \quad \min\ f(x) \quad \text{s.t.} \quad x \in \mathscr{F} = \{x \in R^n | c_i(x) \leqslant 0,\ i \in I;\ c_i(x) = 0,\ i \in L\}, \tag{1}$$

where $I = \{1, 2, \ldots, m\}$, $L = \{m + 1, \ldots, m + p\}$ and $f(x) : R^n \to R$, $c(x) : R^n \to R^{m+p}$ are continuously differentiable functions. A pair $(x, \lambda) \in R^{n+m+p}$ is called a stationary point of (P) if $x \in \mathscr{F}$ and

$$\nabla_x L(x, \lambda) = 0, \quad \lambda_i c_i(x) = 0, \quad i \in I, \tag{2}$$

where $L(x, \lambda) = f(x) + \sum_{i=1}^{m+p} \lambda_i c_i(x)$ is the Lagrange function of (P). Moreover, if $\lambda_i \geqslant 0$, $i \in I$, then (x, λ) is called a KKT point of (P). Sometimes, we also call $x \in \mathscr{F}$ a stationary or KKT point of (P) if there exists $\lambda \in R^{m+p}$ such that (x, λ) is a stationary or KKT point of (P).

Sequential systems of linear equations (SSLE in short) method, or QP-free method, which only requires at each iteration the solution of a few linear systems usually with common coefficient matrices, were developed to address some computational issues in traditional sequential quadratic programming methods (SQPs). For example, the QP subproblems may be inconsistent and the cost for finding their exact solutions can become prohibitive in the absence of a QP truncating scheme. In 1988, Panier et al. [14] proposed a feasible QP-free algorithm for inequality constrained optimization problems, which requires only the solution of two different linear systems and of one linear least-square problem at most iterations. However, in order to prove the uniform nonsingularity of the iteration matrix sequence and the boundedness of the multiplier approximation sequence, they must assume that the strict complementarity holds at all feasible points. Otherwise, the iteration matrix may become ill-conditioned, which will lead to nonconvergence. Moreover, they can only prove the convergence of the iteration to a stationary point while the convergence to a KKT point can be established only if further the number of stationary points is assumed to be finite. The algorithm was later improved by Gao et al. [9] in the sense that every accumulation point of the iterates is a KKT point without assuming the isolatedness of the accumulation point. To achieve this, they solve an extra linear system. However, in the proof of convergence, they must assume that the multiplier sequence is bounded, which is impossible if the iteration matrix is ill-conditioned. In order to avoid the ill-conditioning of the iteration matrix, Qi and Qi [17] proposed a feasible QP-free algorithm for inequality constrained optimization problem, based on a nonsmooth equation reformulation of the KKT system (2), by using the Fischer–Burmeister function that is often used in nonlinear complementarity problems (e.g., [5,12]). They proved that the iteration matrix is uniformly nonsingular and the multiplier approximation sequence bounded even if the strict complementarity condition does not hold at all feasible points. Under a mild condition, their algorithm globally converges to a KKT point. However, the superlinear convergence of their algorithm still requires the strict complementarity condition to hold.

The algorithms mentioned above share a common feature: all the inequality constraints must be involved in their subproblems, leading the computation effect to increase significantly when applied to large-scale problems. Concerning this, Yang and Qi [19] proposed a new SSLE algorithm by introducing a concept of working-set. The new algorithm concerns with only constraints in the working set for their subproblem linear systems, while those not in the working set are totally neglected. By using the Facchinei–Fischer–Kanzow identification technique, they prove that the working set is just the accurate active set at KKT points. Numerical experiments show that the algorithm is especially applicable to