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A Schur complement approach for computing subcovariance matrices arising in a road safety measure modelling

Assi N'Guessan[a,*], Claude Langrand[b]

[a]Ecole Polytechnique Universitaire de Lille et Laboratoire de Mathématiques Appliquées CNRS, FRE 2222, Université de Lille 1, 59655 Villeneuve d'Ascq, Cedex, France
[b]Laboratoire de Mathématiques Appliquées CNRS, FRE 2222.- U.F.R. de Mathématiques pures et appliquées, Université de Lille 1, 59655 Villeneuve d'Ascq, Cedex, France

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Abstract

This paper deals with the determination of the analytical expression of a real partitioned matrix inverse. A formal inversion method is suggested through Schur complements and the elements of any inverse matrix block are explained, irrespective of the size of the considered matrix. The suggested approach does not need any matrix inversion numerical program. This methodology is applied to a Fisher information matrix relating to a multidimensional modelling of road accident data when a road safety measure is applied on different experiment sites. An example of formal calculation and interpretation is given to support our approach.

Keywords: Matrix approximation; Schur complement; Fisher information matrix; Asymptotic covariance matrix; Formal estimation; Road safety measure; Accident data; Multinomial model; Restricted maximum likelihood

1. Introduction

Most statistical studies involving data collecting (random phenomenon modelling, experiment planning, opinion poll, etc.) not only bring out the problems of parameter estimation (looking for optimal solutions) but also those related to the evaluation of the accuracy of those estimations. In many of the

* Corresponding author. Department of Statistics and Computer Sciences, Bât. Polytech'Lille, Université de Lille 1, 59655 Villeneuve d'Ascq, France. Tel.: +33 3 28 76 74 57; fax: +33 3 28 76 73 01.

E-mail addresses: assi.nguessan@polytech-lille.fr (A. N'Guessan), claude.langrand@univ-lille1.fr (C. Langrand).

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statistical evaluation problems, particularly in multivariate statistics, interest parameters are not functionally independent, which means there are relations—constraints—between them. These constraints add further difficulties in bringing out the solutions (estimations) and their accuracy. One of the most popular—as well as the most frequently used—statistical tools to overcome those difficulties still used is the inversion of the so-called Fisher information matrix. This matrix definition uses both the notion of partial derivative of the logarithm of the likelihood function and the linear operator called mathematical expectancy, a notion to which we will come back later in our paper. The concept of the Fisher information matrix is well known and of capital importance in the theory of the parameter statistical estimation (see for example [4,7,9,10,18,22]) and even in the theory of linear systems (see for example [17,20]). Indeed this matrix intervenes in many estimation iterative algorithms, with or without constraints, and its inverse, called asymptotic variance and covariance matrix, generates a measure of the estimation accuracy and of the degree of the linear relations which may exist among the different parameters of the considered model.

In this paper, we focus on the analytical expression of the elements of some blocks of the inverse of a particular Fisher information matrix arising from a statistical model analysis of a road safety measure set up by N'Guessan et al. (see [13,16]). There are certainly other methods (for example [6,23,24,27]) to reach this expression but the one we suggest is based on the Schur complement approach [2,8,19,26] and enables us to get the formal expression of the desired elements. This work stems from N'Guessan's one [14]. Our results go further than those of the latter author and explain the analytical structure of the elements of any block of the inverse of the information matrix considered, irrespective of the size of this matrix. The paper is thus organised. Section 2 states all the work notations and hypotheses used thereafter. Section 3 presents the main technical results and sketches their demonstrations. Section 4 offers an illustrative example coming from the multidimensional modelling of a safety road and accident risk measure. We conclude with an appendix describing the mechanism to obtain the analysed Fisher information matrix structure.

2. Notations and assumptions

Let us consider $s\,(s>0)$ and $r\,(r>1)$ two given integers; n_k $(n_k>0)$ a given integer, $k=1,2,\dots,s$; $z_k=(z_{1k},z_{2k},\dots,z_{rk})^{\mathrm{T}}$ a $r\times 1$ vector of given real data with $z_{jk}>0$, $k=1,2,\dots,s$; $j=1,2,\dots,r$; and note, $\alpha=(\beta_0,\beta^{\mathrm{T}})^{\mathrm{T}}$ a $(1+sr)\times 1$ unknown vector parameter, where $\beta_0(\beta_0>0)$ is a real number, $\beta=(\beta_1^{\mathrm{T}},\beta_2^{\mathrm{T}},\dots,\beta_s^{\mathrm{T}})^{\mathrm{T}}\in\mathbb{R}^{sr}$, with $\beta_k=(\beta_{1k},\beta_{2k},\dots,\beta_{rk})^{\mathrm{T}}$ a $r\times 1$ vector and $\beta_{jk}>0$; for $k=1,2,\dots,s$ $\gamma_k=n_k/(1+\beta_0\langle z_k,\beta_k\rangle)$ a real number, where $\langle,\rangle$ is the usual inner product,

$$V_{\alpha,k}=\frac{\beta_0\gamma_k}{n_k}(z_{1k},z_{2k},\dots,z_{rk})^{\mathrm{T}},$$

a $r\times 1$ vector,

$$\Omega_{\alpha,k}=\operatorname{diag}\left(\frac{1+\beta_0 z_{1k}}{\beta_{1k}},\frac{1+\beta_0 z_{2k}}{\beta_{2k}},\dots,\frac{1+\beta_0 z_{rk}}{\beta_{rk}}\right),$$

a $r\times r$ matrix, $B_{\alpha,k}=\gamma_k(\Omega_{\alpha,k}-V_{\alpha,k}V_{\alpha,k}^{\mathrm{T}})$, a $r\times r$ matrix.