Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Journal of Computational and Applied Mathematics 177 (2005) 467–473

JOURNAL OF
COMPUTATIONAL AND
APPLIED MATHEMATICS

www.elsevier.com/locate/cam

Letter to the Editor

On positive solutions in a phytoplankton–nutrient model[☆]

N.N. Pham Thi[*]

Center for Mathematics and Computer Science (CWI), P.O. Box 94079, 1090 GB, Amsterdam, The Netherlands

Received 4 October 2004; received in revised form 18 October 2004

Abstract

On the basis of an application from aquatic ecology, we discuss the behaviour of the widely used time integration package VODE by Brown et al. (SIAM J. Sci. Statist. Comput. 10 (1989) 1038). When used in a default setting this code smoothly produces a negative steady state solution, which is not realistic in this application.

MSC: Primary 65M12; 65M20

Keywords: Phytoplankton dynamics; Integro-advection–diffusion–reaction equations; Time integration; Positivity

1. Introduction

Phytoplankton, the generic name of microorganisms living in lakes, seas and oceans [8], are at the basis of the aquatic foodweb. Their role for a proper functioning of the aquatic ecosystem has been recognized for a long time and has been widely studied both empirically [17] as well as theoretically [9,5].

For their primary production of biomass, phytoplankton use photosynthesis [12], a process where solar energy (light) and carbon dioxide are utilized. Due to the sequestration of carbon dioxide, phytoplankton have a significant impact on the reduction of the greenhouse effect on a global scale (see e.g. [6]).

[☆] The investigations were supported by the Computational Science Program, which is subsidized by the Netherlands Organization for Scientific Research (NWO). This work was carried out within subtheme MAS1.1—Applications from the Life Sciences.

[*] Tel.: +31 20 592 4077; fax: +31 20 592 4199.

E-mail address: N.N.Pham.Thi@cwi.nl (N.N. Pham Thi).

0377-0427/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2004.11.020

In many regions (and some seasons) light availability is the major factor limiting phytoplankton growth [18]. In other regions, and seasons, phytoplankton growth is largely influenced by the availability of nutrients, such as nitrogen, phosphorous, and iron (see [16,1,3]).

In this note we consider a model in which both limiting factors, light and nutrient, are taken into account. These two factors give rise to contrasting gradients since light is coming from above, whereas nutrients are supplied at the sediment (see e.g. [13,19,4]). As a result, the vertical distribution of the phytoplankton population can be quite heterogeneous in the sense that a large aggregation of phytoplankton is formed at a subsurface depth, where both light and nutrient are just sufficiently available to sustain a population.

2. The mathematical model

Here, we describe the phytoplankton–nutrient model for one single species (a multi-species extension of the model can be found in [15]). The mono-species formulation is sufficient for the purpose of this note: showing the peculiar behaviour of the time integrator VODE [2].

We consider a water column in which the depth co-ordinate z runs from $z = 0$ (the surface) to $z = z_{\mathrm{B}}$ (the bottom). Furthermore, let $\omega(z, t)$ denote the population density of a phytoplankton species at vertical position z at time $t \geqslant 0$. The distribution of phytoplankton is determined by the combined effect of growth (the main biological factor) and local transport processes (the main physical factor) through the partial differential equation

$$\frac{\partial \omega}{\partial t} = g\omega - \frac{\partial J}{\partial z}, \tag{1}$$

where g and J are, respectively, the growth rate and the flux at depth z at time t.

The flux J is determined by the convective transport, due to the settling speed v and the diffusive transport, due to mixing,

$$J(z, t) = v\omega(z, t) - D(z)\,\frac{\partial \omega}{\partial z}(z, t), \tag{2}$$

where $D(z)$ is the space-dependent mixing rate.

In our model, the growth rate g is assumed to depend on the light intensity $\mathscr{L}$ and the nutrient concentration $\mathscr{N}$. In fact, it depends on the balance between the production rate p and the specific loss rate ℓ as given by

$$g(\mathscr{L}, \mathscr{N}) = p(\mathscr{L}, \mathscr{N}) - \ell. \tag{3}$$

Here, the loss rate is assumed constant and represents grazing by zooplankton, mortality, excretion, etc. The production rate p determines the growth of phytoplankton and is defined by the two limiting environmental resources (i.e., light and nutrient) in the following way (see e.g. [13,19]),

$$p(\mathscr{L}, \mathscr{N}) = \mu \min\left(\frac{\mathscr{L}}{L_{\mathrm{H}} + \mathscr{L}}, \frac{\mathscr{N}}{N_{\mathrm{H}} + \mathscr{N}}\right), \tag{4}$$

where μ, L_{H} and N_{H}, respectively, denote the maximum specific production rate and the half-saturation constants of light and nutrient.