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Solution of the Falkner–Skan equation by recursive evaluation of Taylor coefficients

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Abstract

We present a computational method for the solution of the third-order boundary value problem characterized by the well-known Falkner–Skan equation on a semi-infinite domain. Numerical treatments of this problem reported in the literature thus far are based on shooting and finite differences. While maintaining the simplicity of the shooting approach, the method presented in this paper uses a technique known as automatic differentiation, which is neither numerical nor symbolic. Using automatic differentiation, a Taylor series solution is constructed for the initial value problems by calculating the Taylor coefficients recursively. The effectiveness of the method is illustrated by applying it successfully to various instances of the Falkner–Skan equation.

Keywords: Third-order two-point boundary value problem; Shooting; Automatic differentation; Recursive evaluation of Taylor coefficients

1. Introduction

The Falkner–Skan equation describes a nonlinear, one-dimensional third-order boundary value problem, whose solutions are the similarity solutions of the two-dimensional incompressible laminar boundary layer equations. No closed-form solutions are available for this two-point boundary value problem.

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We wish to compute the solution of the Falkner–Skan equation given by

$$\frac{d^3 f}{d\eta^3} + \beta_0 f \frac{d^2 f}{d\eta^2} + \beta\left[1 - \left(\frac{df}{d\eta}\right)^2\right] = 0, \quad 0 < \eta < \infty, \tag{1}$$

subject to the boundary conditions

$$f = 0, \quad \text{at } \eta = 0, \tag{2}$$

$$\frac{df}{d\eta} = 0, \quad \text{at } \eta = 0, \tag{3}$$

$$\frac{df}{d\eta} = 1, \quad \text{as } \eta \to \infty, \tag{4}$$

where β_0 and β are constants. Eq. (1), introduced by Falkner and Skan [8], has been researched extensively in many of its forms, as characterized by varying the values of β_0 and β.

The purpose of this paper is to develop a shooting algorithm which is much more straightforward and simpler than the existing algorithms and which requires much less computational effort. A shooting algorithm for the solution of (1)–(4) involves solving repeatedly the Initial Value Problem (IVP) described by (1)–(3), along with the condition

$$\frac{d^2 f}{d\eta^2} = \alpha, \quad \text{at } \eta = 0. \tag{5}$$

Since different values of α in (5) will lead to different values of $df/d\eta$ as $\eta \to \infty$, we seek that value of α which will yield an f that satisfies (4). The value of α is used to characterize solutions of different instances of the Falkner–Skan equation, and it will be used to compare the results of the present method to those reported previously in the literature.

Previous work on this equation includes the mathematical treatments due to Weyl [24], Coppel [7], and Rosenhead [18]. These works have mainly focused on obtaining existence and uniqueness results. The most significant of these works is that of Coppel [7] in which an elegant proof of the existence, uniqueness, and detailed analyses of solutions to the Falkner–Skan equation, for $\beta \geqslant 0$, with more general initial conditions are given. In addition, Coppel also shows that $\alpha = d^2 f(0)/d\eta^2$ is an increasing function of β. This work has been extended for $\beta < 0$ by Veldman and Van der Vooren [23]. Some new existence and uniqueness results for the solution of the Falkner–Skan equations are also given in [15].

The first computational treatment of the problem was presented in [10]. Smith [22], Cebeci and Keller [6], and Na [14] have considered other numerical treatments. These approaches have used shooting and invariant imbedding. Finite-difference methods for this problem are presented in [2,3]. A differential transformation method, which obtains a series solution of the Falkner–Skan equation is presented in [12]. A new approach to solving this problem by shooting from ∞ (instead of from 0), using some simple analysis of the asymptotic behavior of the solution at ∞, is presented in [21]. Salama [19] develops a one-step method of order 5.

The strength of the method of this paper comes from the fact that it does not use numerical differentiation or other approximations for the derivatives involved in the calculations. It uses a technique known as automatic differentiation, which is neither numerical nor symbolic, and which computes exact derivatives using recursive formulas.