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On the forced oscillation of solutions for systems of impulsive parabolic differential equations with several delays☆

Wei Nian Li[a, b, c, *]

[a] *College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao, 266510, China*
[b] *Department of Mathematics, Shanghai Jiaotong University, Shanghai 200240, China*
[c] *Department of Mathematics, Binzhou University, Shandong 256600, China*

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Abstract

In this paper, we study the forced oscillation of certain systems of impulsive parabolic differential equations with several delays. Some oscillation criteria are established.

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Keywords: Oscillation; Impulsive; System; Parabolic differential equation; Delay

1. Introduction

It is well known that many evolution processes experience changes of state abruptly because of short-term perturbations. We usually regard these perturbations as impulsive type because the duration of these perturbations is negligible in comparison with the duration of the processes considered. In the past few years, the theory of impulsive partial differential equations has been investigated extensively. For instance,

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* Corresponding author. College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao 266510, China.
E-mail address: wnli@263.net (W.N. Li).

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see [1–9] and the references therein. Recently, the oscillations for impulsive delay parabolic differential equations and impulsive delay hyperbolic differential equations were studied by Fu et al. [6] and Cui et al. [3], respectively. But nobody studied the forced oscillation of systems of impulsive delay partial differential equations, as far as we know.

In this paper, we study the forced oscillation of systems of impulsive parabolic differential equations with several delays of the form

$$\begin{aligned}\frac{\partial}{\partial t}u_i(x,t) = {} & \sum_{k=1}^{m} a_{ik}(t)\,\Delta u_k(x,t) + \sum_{k=1}^{m} b_{ik}(t)\,\Delta u_k(x,t-\tau_{ik}) \\ & - c_i(x,t,(u_k(x,t))_{k=1}^m,(u_k(x,t-\sigma_{ik}))_{k=1}^m) \\ & - \sum_{h=1}^{l} q_{ih}(x,t)u_i(x,t-\lambda_{ih}) + f_i(x,t), \quad t \neq j,\end{aligned}$$

$$\begin{aligned}& u_i(x,t_j^+) - u_i(x,t_j^-) = p_i(x,t_j,u_i(x,t_j)), \\ & i \in I_m, \quad j \in I_\infty, \quad (x,t) \in \Omega \times R_+ \equiv G, \end{aligned} \tag{1}$$

where $I_m = \{1,2,\ldots,m\}$, $I_\infty = \{1,2,\ldots\}$, $R_+ = [0,\infty)$, Ω is a bounded domain in R^n with a smooth boundary $\partial\Omega$,

$$\Delta u_i(x,t) = \sum_{r=1}^{n} \frac{\partial^2 u_i(x,t)}{\partial x_r^2}, \quad i \in I_m, \quad 0 < t_1 < t_2 < \cdots < t_j < \cdots$$

and $\lim_{j\to\infty} t_j = \infty$.

Consider the following boundary condition:

$$\frac{\partial u_i(x,t)}{\partial N} = \psi_i(x,t), \quad (x,t) \in \partial\Omega \times R_+, \quad t \neq t_j, \quad i \in I_m, \quad j \in I_\infty \tag{2}$$

and the initial condition

$$u_i(x,t) = \phi_i(x,t), \quad (x,t) \in \Omega \times [-\delta_i, 0], \tag{3}$$

where N is the unit exterior normal vector to $\partial\Omega$ and $\psi_i \in \mathrm{PC}[\partial\Omega \times R_+, R]$, $i \in I_m$, PC denotes the class of functions, which are piecewise continuous in t with discontinuities of first kind only at $t = t_j$ and left continuous at $t = t_j$, $j \in I_\infty$,

$$\delta_i = \max\{\tau_{ik}, \sigma_{ik}, \lambda_{ih}; k \in I_m, h \in I_l\}, \phi_i \in C^2(\Omega \times [-\delta_i, 0], R), i \in I_m, I_l = \{1,2,\ldots,l\}.$$

Throughout this paper, we assume that the following conditions hold:

(C1) $a_{ik}, b_{ik} \in \mathrm{PC}[R_+, R_+]$, $i,k \in I_m$;
(C2) $\tau_{ik} \geqslant 0$, $\sigma_{ik} \geqslant 0$, and $\lambda_{ih} \geqslant 0$ are constants, $i,k \in I_m$, $h \in I_l$;
(C3) $c_i \in \mathrm{PC}[\overline{G} \times R^{2m}, R]$ and

$$c_i(x,t,\xi_1,\ldots,\xi_i,\ldots,\xi_m,\eta_1,\ldots,\eta_i,\ldots,\eta_m) \begin{cases} \geqslant 0 & \text{if } \xi_i \text{ and } \eta_i \in (0,\infty), \\ \leqslant 0 & \text{if } \xi_i \text{ and } \eta_i \in (-\infty,0), \quad i \in I_m; \end{cases}$$