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On convergence of two-stage splitting methods for linear complementarity problems

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Abstract

In this paper, we study the splitting method and two-stage splitting method for the linear complementarity problems. Convergence results for these two methods are presented when the system matrix is an H-matrix and the splittings used are H-splitting. Numerical experiments show that the two-stage splitting method has the same or even better numerical performance than the splitting method in some aspects under certain conditions.

Keywords: Linear complementarity problem; H-matrix; Two-stage splitting method; Convergence theory

1. Introduction

Consider the linear complementarity problems which is abbreviated as LCP(q, M): Find $z \in \mathbb{R}^n$ such that

$$Mz + q \geqslant 0, \quad z \geqslant 0, \quad z^{\mathrm{T}}(Mz + q) = 0,$$

where $M = (m_{ij}) \in \mathbb{R}^{n \times n}$ and $q = (q_i) \in \mathbb{R}^n$ are given real matrix and vector, respectively. This problem arises in various scientific computing areas such as the Nash equilibrium point of a bimatrix game, contact problems, the free boundary problem for journal bearings, etc., see [7].

Over the years, many methods for solving the LCP(q, M) have been developed, see [6,11,13,16]. Most of the methods have their origin in the solution of linear systems and may be classified into two

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categories, pivoting methods and iterative methods. Iterative methods, which generate an infinite sequence converging to a solution of the problem, are particularly effective for large and sparse problems. Recently, much attention has been paid on the class of iterative methods called the splitting method, which is an extension of the matrix splitting method for solving linear systems. Cottle et al. [7] presented detailed descriptions about these methods, and the interested readers may refer to it. In [7] they studied the convergence of the splitting method when matrix M is symmetric or nonsymmetric. Besides, they also developed a two-stage splitting method for the solution of the problem which is mainly based on the idea of inexact iterative methods. But the convergence results they presented for this method are only about symmetric matrices. On the other hand, Machida [12], Bai [1–5] studied the multi-splitting method for solving the LCP(q, M) which are useful in parallel computing. The results they achieved are related to either symmetric matrices or nonsymmetric matrices.

In this paper, we will further study the convergence of the splitting method and the two-stage splitting method. We focus on the nonsymmetric case, particularly the H-matrix case. The results we get for the splitting method generalize and simplify the results in [7] and the results we get for the two-stage splitting method extend the results in [7] to H-matrices which need not be symmetric.

In the following paper, we first present some basic concepts, definitions and some well-known results which shall be used later. Then, in Section 3, we will focus on the splitting method and present some convergence results for this method when the coefficient matrix is an H-matrix. The results obtained are then extended to the two-stage splitting method in Section 4. Numerical experiments are provided in Section 5, which show that two-stage splitting method has the same or even better numerical performance in some aspects comparing with splitting method under certain conditions.

2. Preliminaries

In this section, we briefly introduce some notation, definitions and basic results to be used later.

Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ in the section. By $\rho(A)$ we denote the spectral radius of matrix A. We say that matrix A is convergent if $\rho(A) < 1$. We say that a vector x is nonnegative, denoted by $x \geqslant 0$, if all its entries are nonnegative. Define $x > 0$ if $x \geqslant 0$ with each component $x_i \neq 0$. Similarly, a matrix A is said to be nonnegative, denoted by $A \geqslant 0$, if all its entries are nonnegative or, equivalently it leaves invariant the set of all nonnegative vectors. For two matrices A and B of the same size, we say $A \geqslant B$ $(A > B)$ when $A - B \geqslant 0$ $(A - B > 0)$. We define $|A| = (|a_{ij}|)$, and this symbol also applies to vectors. By I_m we denote the $m \times m$ identity matrix and when the order of the identity matrix is clear from the context, we simply denote it by I.

Let $\mathbb{Z}^{n\times n}$ denote the set of all real $n \times n$ matrices which have all nonpositive off-diagonal entries. A nonsingular matrix $A \in \mathbb{R}^{n\times n}$ is called monotone matrix if $A^{-1} \geqslant 0$; A nonsingular matrix $A \in \mathbb{Z}^{n\times n}$ is called M-matrix if $A^{-1} \geqslant 0$. For any matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, we define its comparison matrix $\langle A\rangle = (\alpha_{ij})$ by $\alpha_{ii} = |a_{ii}|$ and $\alpha_{ij} = -|a_{ij}|$, $i \neq j$. Furthermore, A is said to be a H-matrix if $\langle A\rangle$ is an M-matrix, i.e., $\langle A\rangle^{-1} \geqslant 0$. Of course, M-matrices are special cases of H-matrices. H-matrices are always nonsingular but, in contrast to M-matrices, H-matrices need not be monotone.

A matrix $M \in \mathbb{R}^{n\times n}$ is called a Q-matrix if the LCP(q, M) has a solution for any $q \in \mathbb{R}^n$, and called a P-matrix if all its principle minors are positive. A matrix M is a P-matrix if and only if the LCP(q, M) has a unique solution for all vectors $q \in \mathbb{R}^n$. Clearly, a P-matrix is a Q-matrix. The following result is often used in our paper.