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Near minimally normed spline quasi-interpolants on uniform partitions

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Abstract

Spline quasi-interpolants (QIs) are local approximating operators for functions or discrete data. We consider the construction of discrete and integral spline QIs on uniform partitions of the real line having small infinity norms. We call them near minimally normed QIs: they are exact on polynomial spaces and minimize a simple upper bound of their infinity norms. We give precise results for cubic and quintic QIs. Also the QI error is considered, as well as the advantage that these QIs present when approximating functions with isolated discontinuities.

Keywords: B-splines; Discrete quasi-interpolants; Integral quasi-interpolants; Infinity norm

1. Introduction

Usually, the construction of spline approximants requires the solution of linear systems. Spline quasi-interpolants (QIs) are local approximants avoiding this problem, so they are very convenient in practice. For a given nondecreasing biinfinite sequence $\mathbf{t}=(t_i)_{i\in\mathbb{Z}}$ such that $|t_i| \to +\infty$ as $i \to \pm\infty$, and $t_i < t_{i+k}$ for all i, let $N_{i,k}$ be the ith B-spline of order $k \in \mathbb{N}$, and $S_{k,\mathbf{t}}$ the linear space spanned by these B-splines (see e.g. [28]). In [4,6], QIs $Qf = \sum_{i\in\mathbb{Z}} \lambda_i(f)N_{i,k}$ were constructed, where the linear form λ_i uses both functional and derivative values (see [5, Chapter XII; 13, Chapter 5; 28, Chapter 6], as well as [21,27] for

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the uniform case). In [17], another B-spline QI method is defined with λ_i involving functional values in a divided difference scheme. A discrete QI (dQI) is obtained when $\lambda_i(f)$ is a finite linear combination of functional values. This problem has been considered in [10] (see also [21–24]). When $\lambda_i(f)$ is the inner product of f with a linear combination of B-splines, Q is an integral QI (iQI). For details on this topic, see for example [12,14,20,24–26] (and references therein).

All these QIs are exact on the space $\mathbb{P}_{k-1}$ of polynomials of degree at most $k-1$ (or a subspace of $\mathbb{P}_{k-1}$). In [16], one can find a general construction of univariate spline QIs reproducing the spline space $S_{k,\mathbf{t}}$.

Once the QI is constructed, a standard argument (see e.g., [13, p. 144]) shows that, if $\mathscr{S}$ is the reproduced space, then $\|f - Qf\|_\infty \leqslant (1 + \|Q\|_\infty)\mathrm{dist}(f, \mathscr{S})$. This leads us to the construction of QIs with minimal infinity norm (see [7, p. 73, 21] in the box-spline setting). This problem has been considered in the discrete case in [1,15].

Here, we are interested in spline QIs Q on uniform partitions of the real line such that (a) $\lambda_i(f) := \lambda(f(\cdot + i))$ is either a linear combination of values of f at points lying in a neigbourhood of the support of the ith B-spline, or the inner product of f with a linear combination of B-splines; (b) Q reproduces the polynomials in the spline space; and (c) a simple upper bound of the infinity norm of Q is minimized.

The paper is organized as follows. In Section 2, we establish the exactness conditions on polynomials of the dQIs or iQIs. In Section 3, we define a minimization problem whose solution will be called near minimally normed (NMN) dQI or iQI. In Sections 4 and 5, we describe some NMN cubic and quintic dQIs and iQIs respectively. In Section 6, we establish some error bounds for cubic dQIs and iQIs. In Section 7, we show that these QIs diminish the overshoot when applying them to the Heaviside function, so seem suitable for the approximation of functions with isolated discontinuities.

Only the even order B-splines are considered here, because the results for the others are similar.

2. Discrete and integral QIs on uniform partitions

Consider the sequence $\mathbf{t} = \mathbb{Z}$ of integer knots. Let $M := M_{2n}$ be the B-spline of even order $2n$, $n \geqslant 2$, with support $[-n, n]$ (see e.g., [27,28]). We deal with discrete and integral spline QI operators

$$Qf = \sum_{i \in \mathbb{Z}} \lambda_i(f) M_i,$$

where $M_i := M(\cdot - i)$, and the linear form λ_i has one of the two following forms

(i) $\lambda_i(f) = \sum_{j=-m}^{m} \gamma_j f(i-j)$ when Q is a dQI, and
(ii) $\lambda_i(f) = \sum_{j=-m}^{m} \gamma_j \langle f, M_{i-j} \rangle$ when Q is an iQI, with $\langle f, g \rangle := \int_{\mathbb{R}} fg$

for $m \geqslant n$ and $\gamma_j \in \mathbb{R}$, $-m \leqslant j \leqslant m$.

Defining the fundamental function

$$L := L_{2n,m} = \sum_{j=-m}^{m} \gamma_j M_j, \tag{1}$$