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Monotonicity preserving rational spline histopolation

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Abstract

For given monotone data we propose the construction of a histopolating linear/linear rational spline of class C^1. The uniqueness and existence of this spline is proved. The method is implemented via the representation with histogram heights and first derivatives of the spline. The use of Newton's method and ordinary iterations are discussed. Numerical tests support completely the theoretical results.

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1. Introduction

In several practical applications one would like to construct a function S with some smoothness which reflects the shape of the histogram and whose integral over any particular interval is equal to the area of the corresponding histogram rectangle, i.e.

$$\int_{x_{i-1}}^{x_i} S(x)\,\mathrm{d}x = z_i(x_i - x_{i-1}), \quad i = 1, \ldots, n,$$

where z_i is the histogram height over the interval $[x_{i-1}, x_i]$. It is known that, for example, cubic and quadratic polynomial spline histopolants without additional knots do not preserve monotonicity or

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convexity of given data. On the other hand, a linear/linear rational spline of class C^1, being strictly monotone or constant everywhere, cannot histopolate non-monotone data. Thus, it is natural to ask whether there is a linear/linear rational spline histopolant to any monotone data. We will answer to this question in our work. Note that similar problem for interpolation is solved in [5,7].

Another approach in shape preserving approximation is to increase the degree of spline or to use additional knots or parameters. For example, in [11], quadratic/linear rational splines of class C^1 with somewhat special introduction of parameters are studied. For the existence of monotone (or convex) histosplines sufficient and necessary conditions are derived, which always can be satisfied by choosing the parameters appropriately. In [10], having some similarity to [11], cubic/linear rational splines of class C^2 with special choice of parameters are considered. Conditions for the existence of monotone (or convex) histosplines are established and they mean, actually, that the parameters characterizing the rationality have to be sufficiently large. The authors of [8] analyze a C^2 class cubic/linear rational spline interpolation preserving monotonicity. As an application, it is proposed to use the derivatives of rational interpolating splines as monotonicity preserving histopolants. In [4], classical quadratic splines of class C^1 with two additional uniformly spaced knots on each particular interval for the histopolation of monotone data are studied. Conditions for monotonicity (and positivity) concerning the values of splines in additional knots are established. Two algorithms for computing spline parameters are presented and analyzed.

The histopolation algorithms with polynomial splines are presented in [12] where the reader can find also several references to earlier results on this topic.

We refer the reader to [3] for general information about shape preserving approximation, see also the references in [10] for shape preserving interpolation.

2. The histopolation problem

Let x_i be given points in an interval $[a, b]$ such that $a = x_0 < x_1 < \cdots < x_n = b$ and let z_i, $i = 1, \ldots, n$, be given real numbers. We want to construct a C^1 smooth function S on $[a, b]$ of the form

$$S(x) = \frac{a_i + b_i(x - x_{i-1})}{1 + d_i(x - x_{i-1})} \tag{1}$$

with $1 + d_i(x - x_{i-1}) > 0$ for $x \in [x_{i-1}, x_i]$, $i = 1, \ldots, n$ (i.e. a linear/linear rational spline), satisfying the histopolation (area-matching) conditions

$$\int_{x_{i-1}}^{x_i} S(x)\,\mathrm{d}x = z_i(x_i - x_{i-1}), \quad i = 1, \ldots, n. \tag{2}$$

In addition, we impose the boundary conditions

$$S'(x_0) = \alpha, \quad S'(x_n) = \beta \tag{3}$$

or

$$S(x_0) = \alpha, \quad S(x_n) = \beta \tag{4}$$

for given α and β. However, one condition from (3) and another from (4) at different endpoints x_0 and x_n may be used.