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Permanence of a single-species dispersal system and predator survival

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Abstract

This paper considers permanence of a single-species dispersal periodic system with the possibility of the loss for the species during their dispersion among patches. The condition obtained for permanence generalizes the known condition on the system without loss for the species in the process of movement. Next, we add predators into every patch and consider the survival possibility of the predator. It is shown that the total amount of the predators can remain positive, if the single-species (prey) dispersal system has a positive periodic solution and the quantity of prey in each patch is enough for survival of the predator.

MSC: 92D25; 34C60

Keywords: Predator–prey system; Dispersal; Permanence; Periodic solution

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1. Introduction

Since the interrelationship between the organisms and the environment seems to play an essential role on the stability (or permanence) of ecological systems, the effect of the dispersion on the possibility of species survival has been an important subject in population biology.

In the former studies on dispersal population models, the single and predator–prey models described by autonomous ordinary differential equations have long played an important role in mathematical population biology [1–5,10–17,19–24,27]. Recently, some authors have also studied the influence of dispersal on the time dependent population models because some realistic parameters change dependent upon seasonal environment [6–9,25,26,28]. Most of the studies assume that the intrinsic growth rates of the prey species are all continuous and bounded above and below by positive constants (this means that every species lived in a suitable environment). They obtained some sufficient conditions that guarantee permanence of every species and global stability of a unique positive periodic solution.

However, the actual living environment of endangered species is not always like this. Because of the ecological effects of the human activities and industry, e.g., the location of manufacturing industries, the pollution of the atmosphere, of river, of soil, etc., more and more habitats were broken into smaller patches and some of the patches were polluted. In some of these patches the species will go extinct without the contribution from the other patches, and hence the species must live in a poor patchy environment [9].

This paper considers the time-dependent population models with single and predator–prey species which live in a poor patchy environment. At the same time, by using these models we try to describe a kind of biological phenomenon that there are some losses for the species when the individuals move from one patch to another patch.

Let us suppose that the environment is spatially heterogeneous and is partitioned into several subspaces (called patches). The species can disperse among the patches and the dispersion is not safe, that is, there is some possibility for the species to die in the process of movement between patches. Further, we add predators into all patches and assume that the predator is confined in each patch. This is a typical situation like ecological systems composed of several islands. Here, the dispersable species is a kind of birds and its predator is an animal foraging the birds. For the predator, the sea is barrier to dispersal.

In this paper, we consider the above system under the (time) periodic environment. Main problems are as follows: (i) to obtain permanence conditions of prey dispersal periodic systems; (ii) to study survival possibility of the predator under the permanent prey systems.

The organization of the paper is as follows. In the next section, we describe the model and give some useful theorems to study the dynamical properties of the model. In Section 3, we give the main theorems and explain their biological implications. The detailed proofs are given in the final section.

2. The models and preliminaries

We first describe the following single-species system in patchy environment

$$\dot{x}_i = x_i[b_i(t) - a_i(t)x_i] + \sum_{j=1}^{n}(1 - \lambda_{ij}(t))D_{ij}(t)x_j - \sum_{j=1}^{n} D_{ji}(t)x_i, \quad i = 1, 2, \ldots, n, \tag{2.1}$$