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An optimized Runge–Kutta method for the solution of orbital problems

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Abstract

We present a new explicit Runge–Kutta method with algebraic order four, minimum error of the fifth algebraic order (the limit of the error is zero, when the step-size tends to zero), infinite order of dispersion and eighth order of dissipation. The efficiency of the newly constructed method is shown through the numerical results of a wide range of methods when these are applied to well-known periodic orbital problems.

MSC: 65L05; 65L06

Keywords: Dispersion; Dissipation; Orbital problems; Runge–Kutta; Explicit methods

1. Introduction

Many orbital problems in astronomy, astrophysics, celestial mechanics, etc. are expressed by the second-order differential equation of the form

$$\begin{aligned} &y''(x) = f(x, y(x)), \quad y(x_0) = y_0, \\ &y'(x_0) = y'_0, \end{aligned} \tag{1}$$

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that is, differential equations where f is independent from the first derivative of y. In order to use first-order numerical methods to solve these problems we set

$$y_1(x) = y(x),$$
$$y_2(x) = y'(x).$$

In this way, (1) can be expressed by a system of two first-order ODEs

$$\begin{aligned} y_1'(x) &= y_2(x), \\ y_2'(x) &= f(x, y_1(x)). \end{aligned} \tag{2}$$

2. Basic theory

2.1. General form of explicit Runge–Kutta methods

An s-stage explicit Runge–Kutta method used for the computation of the approximation of $y_{n+1}(x)$ in problem (2), when $y_n(x)$ is known, can be expressed by the following relations:

$$\begin{aligned} y_{n+1} &= y_n + \sum_{i=1}^{s} b_i k_i, \\ k_i &= hf\left(x_n + c_i h,\ y_n + h \sum_{j=1}^{i-1} a_{ij} k_j\right), \quad i = 1, \ldots, s. \end{aligned} \tag{3}$$

The method mentioned previously can also be presented using the Butcher table below:

$$\begin{array}{c|ccccc} 0 & & & & & \\ c_2 & a_{21} & & & & \\ c_3 & a_{31} & a_{32} & & & \\ \vdots & \vdots & \vdots & & & \\ c_s & a_{s1} & a_{s2} & \ldots & a_{s,s-1} & \\ \hline & b_1 & b_2 & \ldots & b_{s-1} & b_s \end{array}. \tag{4}$$

The following equations must always hold:

$$c_i = \sum_{j=1}^{s} a_{ij}, \quad i = 2, \ldots, s. \tag{5}$$

Definition 1 (*Hairer et al. [4]*). A Runge–Kutta method has algebraic order p when the method's series expansion agrees with the Taylor series expansion in the p first terms

$$y^{(n)}(x) = y_n^{(n)}(x), \quad n = 1, 2, \ldots, p.$$