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New developments on equivalent thermal in hydrothermal optimization: an algorithm of approximation

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Abstract

In this paper we revise the classical formulation of the problem of the optimization of hydrothermal systems. First we demonstrate that a number of thermal plants can be substituted by a single one that behaves equivalently to the entire set. We then calculate the equivalent plant in the case where the cost functions are general (nonquadratic). We prove that the equivalent thermal plant is a second-order polynomial with piece-wise constant coefficients. Moreover, it belongs to the class C^1. Next we calculate the equivalent plant in the case of imposing constraints of minimum or maximum thermal power. Finally, we present an example and execute the proposed algorithm using Mathematica package.

MSC: 49K30

Keywords: Optimization; Hydrothermal systems; Equivalent thermal

1. Introduction

This paper studies the optimization of hydrothermal systems. A hydrothermal system is made up of hydraulic and thermal power plants that must jointly satisfy a certain demand in electric power during a definite time interval.

The idea of introducing an equivalent thermal plant has already appeared in several earlier studies. In [3] the authors consider it in application to purely thermal problems, though they did not notice the

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need to define the equivalent plant piece-wisely, since the restriction of power positivity is ignored. The idea has also been used in problems with hydraulic components. For example, [7] reports the application of the discrete maximum principle and [5] considers the application of a modified algorithm based on Pontryagin's maximum principle.

The concept of the equivalent thermal plant has been used up until now. Thus, [8] and [9] develop a short-term hydrothermal scheduling algorithm based on the simulated annealing technique, and an efficient short-term hydrothermal scheduling algorithm is proposed in [4] based on the evolutionary programming technique.

In a previous paper [1] we considered the possibility of substituting a problem with m thermal plants and n hydroplants ($\mathrm{H}_{\mathbf{n}} - \mathrm{T}_{\mathbf{m}}$) by an equivalent problem ($\mathrm{H}_{\mathbf{n}} - \mathrm{T}_{\mathbf{1}}$) with a single thermal power station: the equivalent thermal plant. In said paper, we calculated the equivalent minimizer in the case where the cost functions are second-order polynomials. We proved that the equivalent minimizer is a second-order polynomial with piece-wise constant coefficients; moreover, it belongs to the class C^1.

In this paper, we shall add various fundamental contributions. First we continue the theoretical studies of the equivalent thermal plant. We prove the existence and uniqueness of the equivalent minimizer, under certain assumptions. We then calculate the equivalent minimizer for a general (nonquadratic) model and go on to prove that it belongs to the class C^1.

Next we prove that, under certain hypotheses, the existence and uniqueness of the equivalent minimizer is guaranteed in the case of imposing constraints of minimum or maximum thermal power, and we go on to calculate the equivalent plant in this case. Finally, we present an example and perform the proposed algorithm using Mathematica package.

2. Description of the problem

Let us assume that a hydrothermal system accounts for m thermal plants. We assume the following definitions throughout the paper.

Let $F_i : D_i \subseteq \mathbb{R} \to \mathbb{R}$ ($i = 1, \ldots, m$) be the cost functions of the thermal power plants. We assume that

$$\forall \xi \in D = D_1 + \cdots + D_m \subseteq \mathbb{R}, \quad \exists (x_1, \ldots, x_m) \in \prod_{i=1}^{m} D_i$$

the unique minimum of $\sum_{i=1}^{m} F_i(x_i)$ with the condition $\sum_{i=1}^{m} x_i = \xi$.

Definition 1. Let us call the ith distribution function, the function

$$\Psi_i : D_1 + \cdots + D_m \to D_i$$

defined by $\Psi_i(\xi) = x_i, \forall i = 1, \ldots, m$.