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Wavelet-like bases for thin-wire integral equations in electromagnetics

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Abstract

In this paper, wavelets are used in solving, by the method of moments, a modified version of the thin-wire electric field integral equation, in frequency domain. The time domain electromagnetic quantities, are obtained by using the inverse discrete fast Fourier transform. The retarded scalar electric and vector magnetic potentials are employed in order to obtain the integral formulation. The discretized model generated by applying the direct method of moments via point-matching procedure, results in a linear system with a dense matrix which have to be solved for each frequency of the Fourier spectrum of the time domain impressed source. Therefore, orthogonal wavelet-like basis transform is used to sparsify the moment matrix. In particular, dyadic and *M*-band wavelet transforms have been adopted, so generating different sparse matrix structures. This leads to an efficient solution in solving the resulting sparse matrix equation. Moreover, a wavelet preconditioner is used to accelerate the convergence rate of the iterative solver employed. These numerical features are used in analyzing the transient behavior of a lightning protection system. In particular, the transient performance of the earth termination system of a lightning protection system or of the earth electrode of an electric power substation, during its operation is focused. The numerical results, obtained by running a complex structure, are discussed and the features of the used method are underlined.

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1. Introduction

Wavelet theory is a relevant continuously emerging area in mathematical research. It has been applied in a wide range of engineering disciplines and it has received considerable attention in computational electromagnetics, particularly in solving integral equations. With reference to this mathematical problem, it is well-known that these equations are obtained by applying boundary conditions to an integral representation of the electromagnetic (EM) field. In solving complex scattering EM problems, the integral equations approach has the advantage of reducing the solution domain to a small finite region containing the boundary conditions implicitly; but, as a drawback, in this formulation the global interactions among sources, object and fields have to be specified. The numerical method generally employed to solve integral equations is the method of moments (MoM) [21]. In its direct formulation, MoM results in the solution of a linear system. So operating, when scattering problems from electrically large complex objects have to be approached, a dense linear systems have to be treated.

In this paper, wavelets are used in solving, by MoM, a modified version of the thin-wire electric field integral equation (EFIE), in frequency domain. The final aim is the analysis of the transient behavior of a lightning protection system (LPS) [3]. Thus, the time domain EM quantities, are obtained by using the inverse discrete fast Fourier transform (DFFT). In particular in this paper, the transient performance of the earth termination system of an LPS during its operation or of the earth electrode of an electric power substation is focused [1,2,6–8]. The retarded scalar and vector potentials are employed in order to obtain the integral formulation. By applying the direct MoM via point-matching procedure a linear system with dense matrix must be solved for each significant frequency of the Fourier spectrum of the time domain EM source.

The size of the dense linear system grows by increasing the partitioning of the discretized model. Therefore, in order to lead with large scale problems, a matrix sparsification is really important. It has been found that the application of wavelet transform can result in a very sparse moment matrix [4,5,9,10,14,15,17,22]. As a result, the computational complexity associated with the solution of electromagnetic integral equations is significantly reduced, by applying an iterative solver; however, no improvements are reached in the convergence rate. Hence, preconditioners involving different wavelet transforms have been generated, by exploiting the sparsity of the modified MoM matrix.

The paper is organized as follows. In Section 2, the electromagnetic problem and the discretized model are introduced. Section 3 briefly reviews selected preliminaries on wavelet transform referred to the sparsification and preconditioning of MoM matrices. In Section 4, the numerical results obtained by running a complex earthing structure, as those employed in a LPS or in electric power substations, are discussed and the features of the used method are underlined.

2. Problem formulation

In this section an essential description of the EM problem formulation is sketched. More complete analytical developments and physical considerations can be found in [3] to which the reader is invited to refer.

For the whole LPS made up of interconnected cylindrical straight conductors the well-known thin-wire hypothesis can be adopted. As a consequence, the current I_a on each wire, can be represented by a filament placed on its axis. In order to obtain an integral equation for the longitudinal currents, the