Available online at www.sciencedirect.com
SCIENCE DIRECT®
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
Journal of Computational and Applied Mathematics 175 (2005) 113–136
www.elsevier.com/locate/cam

Computing Nash equilibria through computational intelligence methods

N.G. Pavlidis, K.E. Parsopoulos, M.N. Vrahatis*

Department of Mathematics, University of Patras Artificial Intelligence Center (UPAIRC), University of Patras, GR 26110 Patras, Greece

Received 5 October 2003; received in revised form 18 February 2004

Abstract

Nash equilibrium constitutes a central solution concept in game theory. The task of detecting the Nash equilibria of a finite strategic game remains a challenging problem up-to-date. This paper investigates the effectiveness of three computational intelligence techniques, namely, covariance matrix adaptation evolution strategies, particle swarm optimization, as well as, differential evolution, to compute Nash equilibria of finite strategic games, as global minima of a real-valued, nonnegative function. An issue of particular interest is to detect more than one Nash equilibria of a game. The performance of the considered computational intelligence methods on this problem is investigated using multistart and deflection.

MSC: 91A99; 91A05; 91A06; 74P99

Keywords: Nash equilibria; Evolutionary algorithms; Particle swarm optimization; Differential evolution; Evolution strategies

1. Introduction

Game theory is a mathematical theory of socio-economic phenomena exhibiting interaction among decision-makers, called *players*, whose actions affect each other. The fundamental assumptions that underlie the theory are that players pursue well-defined exogenous objectives and take into account

* Corresponding author. Tel.: +30-2610-997374; fax: +30-2610-992965.
E-mail addresses: npav@math.upatras.gr (N.G. Pavlidis), kostasp@math.upatras.gr (K.E. Parsopoulos), vrahatis@math.upatras.gr (M.N. Vrahatis).

0377-0427/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2004.06.005

their knowledge, or expectations, of other players' behavior [17]. The theory has been so far applied in the fields of economics, political science, evolutionary biology, computer science, statistics, accounting, social psychology, law, and branches of philosophy such as epistemology and ethics [1].

A *game* is a model of strategic interaction among a number of players, which includes the constraints on the actions that players can take and the players' interests, but does not specify the actions that players do take. A *solution* is a systematic description of the outcomes that may emerge in a game [17]. In this paper we consider only the family of *strategic,* or *normal form*, games. The most commonly encountered solution concept in game theory is that of *Nash equilibrium* [15,16]. This notion captures a steady state of the play of a strategic game, in which each player holds correct expectations concerning the other players' behavior and acts rationally.

The problem of detecting the Nash equilibria of a finite strategic game admits a number of alternative formulations, yet computing such solutions remains a challenging task up-to-date (for a comprehensive review on algorithms to compute equilibria of *n*-person games see [13], for a survey of algorithms for 2-player games see [28]). Furthermore, an algorithm that computes a single Nash equilibrium is unsatisfactory for many applications. Even if the resulting equilibrium is perfect, or satisfies some other criterion posed in the literature on refinements of Nash equilibrium, we cannot eliminate the possibility that other, more salient equilibria exist [13].

The problem of computing a Nash equilibrium can be formulated as a global optimization problem [12]. This formulation allows us to consider three computational intelligence methods, namely, covariance matrix adaptation evolution strategies (CMA-ES), particle swarm optimization (PSO), and differential evolution (DE), to detect Nash equilibria. CMA-ES, PSO and DE are stochastic optimization methods capable of handling nondifferentiable, nonlinear and multimodal objective functions. They exploit a population of potential solutions to probe the search space synchronously. Each member of the population adapts its position towards the most promising regions of the function's landscape, characterized, in the case of minimization, by lower function values. Incorporating *multistart* [30] or *deflection* [11], more than one global minima of the objective function can be obtained.

The remaining paper is organized as follows: Section 2 is devoted to a brief exposition of basic concepts of game theory and the formulation of the problem. In Section 3, the computational intelligence methods considered, as well as, the techniques employed to compute more than one minimizers of a function, are described. Section 4 outlines the proposed algorithmic scheme and discusses the experimental results. The paper ends with conclusions in Section 5.

2. Notation and problem formulation

2.1. Strategic games and Nash equilibrium

A *finite strategic game*, $\Gamma = \langle(\mathscr{N}), (S_i), (u_i)\rangle$, is defined by [17],

- a finite set $\mathscr{N} = \{1, \ldots, N\}$ of *players*,
- for each player $i \in \mathscr{N}$, a set of actions, pure strategies, $S_i = \{s_{i1}, \ldots, s_{im_i}\}$,
- for each player $i \in \mathscr{N}$, a *payoff function*, $u_i : S \to \mathbb{R}$, is also defined, where $S = S_1 \times S_2 \times \ldots \times S_{\mathscr{N}}$ is the Cartesian product of all sets S_i.