Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Journal of Computational and Applied Mathematics 175 (2005) 137–147

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.com/locate/cam

A fourth algebraic order trigonometrically fitted predictor–corrector scheme for IVPs with oscillating solutions

G. Psihoyios[a], T.E. Simos[b,1,*]

[a]*Department of Mathematics and Technology, School of Applied Sciences, Anglia Polytechnic University, East Road, Cambridge CB1 1PT, UK*

[b]*Department of Computer Science and Technology, Faculty of Sciences and Technology, University of Peloponnese, GR-22 100 Tripolis, Greece*

Received 1 October 2003; received in revised form 25 February 2004

Abstract

A scheme of trigonometrically fitted predictor–corrector (P–C) Adams–Bashforth–Moulton methods is constructed in this paper. Our new P–C method is based on the third order Adams–Bashforth scheme (as predictor) and on the fourth order Adams–Moulton scheme (as corrector). We tested the efficiency of our newly developed scheme against well known methods, with excellent results. The numerical experimentation showed that our method is considerably more efficient compared to well known methods used for the numerical solution of initial value problems with oscillating solutions.

MSC: 65L05; 65L06

Keywords: Numerical solution; Initial value problems (IVPs); Adams–Bashforth methods; Adams–Moulton methods; Predictor–corrector (P–C) methods; Trigonometric fitting; Multistep methods

* Corresponding author. 26 Menelaou Street, Amfithea - Paleon Faliron, GR-175 64 Athens, Greece. Tel./fax: +30-210-942-0091.

E-mail addresses: g.y.psihoyios@apu.ac.uk (G. Psihoyios), tsimos@mail.ariadne-t.gr (T.E. Simos).

URL: http://www.uop.gr/~simos.

[1] Active Member of the European Academy of Sciences and Arts; Visiting Professor, Department of Mathematics, Anglia Polytechnic University.

0377-0427/$ - see front matter
doi:10.1016/j.cam.2004.06.014

1. Introduction

Equations or systems of equations of the form

$$\mathbf{y}'(\mathbf{x}) = \mathbf{f}(\mathbf{x}, \mathbf{y}), \quad \mathbf{y}(\mathbf{x}_0) = \mathbf{y}_0 \tag{1}$$

are used as the mathematical model for problems in celestial mechanics, physical chemistry and chemical physics, quantum mechanics, electronics, nanotechnology, materials sciences and elsewhere. Special attention deserve the class of the above equations with oscillatory/periodic solution (see [11,13]).

The investigation of the numerical solution of the above equation is the subject of extensive research activity during the last two decades (indicatively see [1–3,8,9,12,17–19,21–23,26,28,29] and references therein). One may also find extensive reviews about the methods developed for the solution of (1) with oscillating behavior in [8,21,24] and references therein, Quinlan and Tremaine [17], as well as [25,28,29]. A main characteristic of all the methods developed in the literature for the numerical solution of (1) is that they belong to the class of multistep and hybrid techniques. Moreover, the majority of the these methods have been designed for the numerical solution second order differential equations.

One of the better processes for the development of efficient methods for the numerical integration of first order initial value problems with oscillating or periodic solution is the exponential and the trigonometric fitting technique first introduced by Lyche [12]. Raptis and Allison [19] then developed a much more efficient than the original, Numerov type exponentially fitted method for second order differential equations arising from the Schrödinger equation. More recently, Van Daele et al. [27] used a quite different methodology to ours in order to introduce trigonometric fitting to the so called *r-Adams* methods. We have also applied trigonometric fitting to very different types of predictor–corrector (P–C) methods, like the Explicit Advanced Step-point or EAS methods (see [15]), which had been introduced in their non-trigonometrically fitted form in [14].

In this work we use trigonometric fitting in order to construct an Adams–Bashforth–Moulton fourth algebraic order (P–C) scheme, which will be employed to solve first order IVPs with oscillating solution. In a previous paper of ours [16], we had applied trigonometric fitting to a lower algebraic order P–C scheme with excellent results.

We have structured this paper as follows: In Section 2 we develop the new trigonometrically fitted method. In Section 3 we investigate and briefly discuss the stability of our new scheme. In Section 4 we proceed to the numerical illustrations. Finally, in Section 5 we present the concluding remarks.

2. Trigonometrically fitted fourth algebraic order P–C scheme

The P–C family of methods appearing below has been widely used (e.g. by Shampine and Gordon [20])

$$\begin{aligned}\overline{y}_{n+1} &= y_n + h\sum_{i=0}^{k-1} b_i \nabla^i f_n,\\ y_{n+1} &= y_n + h\sum_{i=0}^{k} \beta_i \nabla^i \overline{f}_{n+1}.\end{aligned} \tag{2}$$