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On analogy and dissimilarity of dependence of stability on several parameters in flow simulations

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Abstract

In the present paper, we investigate the asymptotic behavior of numerical solutions in direct fluid simulations. The incompressible Navier–Stokes equations and the continuity equation are solved numerically by using the marker-and-cell method. The model adopted in the present study is a flow around a two-dimensional circular cylinder. Dependence of the unsteady structure of numerical solutions on several parameters are discussed by analyzing the behavior of numerical drag coefficient C_{d}. Concretely, we concentrate the dependence of bifurcation processes on the amplitude of second- and fourth-order viscosity terms and time increment. Though the numerical fourth-order artificial viscosity has the stabilizing effect like the physical second-order one, the bifurcation processes are different. Furthermore, it is clarified that adopting small time increment values does not always produce the reasonable results.

Keywords: Numerical approximation of solutions; Finite difference methods; Navier–Stokes equations; Asymptotic behavior of solutions; Bifurcation

1. Introduction

Recent development of supercomputer has made it possible to simulate complicated unsteady flow fields. Though a lot of computational studies concerning the complicated unsteady flow structure have been reported in many references, it has been left unknown whether numerical solutions given in those contributions correspond to the true solutions of the original fluid equations or not. On the other hand, it is well known that the complicated flow structure also appears in the cases of the low Reynolds numbers, where the flows are expected to be laminar physically, under some

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conditions. The approximate solutions which do not correspond to the true one are often called spurious solutions or ghost ones and they have been studied in detail in [6,12]. When the differential equations are discretized, a lot of terms and parameters which are not included in the original differential equations are added. Yamaguchi and co-researchers [8,11] analytically studied about the ghost solutions in detail and concluded that the discretization induces instability or stability. We also studied the characteristic behavior of the numerical solutions after the transients of simple scalar nonlinear differential equations [2,3] and concluded that we may get ghost solutions, even if we adopt more accurate schemes. Furthermore, these ghost solutions also appear in the practical numerical simulations for flow fields. In most cases, the approximate conditions of calculations are set to be physically reasonable and the most suitable schemes to obtain reasonable results are selected. However, the complicated flow structure appears even in the cases of the low Reynolds numbers by mistaking several numerical conditions. For example, it is expected that adopting large Δt values causes the instability which is similar to those in the cases of calculations of high Reynolds number. There is a possibility that a parameter like the large Δt may play the similar role to another one like the large Reynolds number.

By the way, the artificial viscosity terms which are added explicitly and implicitly in order to stabilize the system are often utilized in the direct numerical flow simulations. Needless to say, we cannot perform calculations stably without these stabilizing terms in the cases of the high Reynolds number because the nonlinear instability grows and calculations diverge. In our previous reports, we also discussed about the nonlinear structure of the numerical solutions after the transients calculated by using several kinds of upwind schemes [4,5]. In the case of the upwind spatial discretization, no spurious numerical solution appeared in some cases. However, we got several types of complicated spurious solutions in other cases even when we used the high-order accurate upwind schemes which included high-order artificial viscosity terms. In these cases, the added terms sometimes bring other types of nonlinear instabilities and the qualitative structure of the numerical solutions after the transients changes. In other words, there is a possibility for us to get multiple stable and unstable ghost solutions under the same original physical conditions and parameters.

One of purposes of the present work is to investigate the characteristics of these ghost solutions from the viewpoints of the comparison of qualitative structure of instability induced by the numerical fourth-order artificial viscosity terms with that induced by the physical second-order viscosity terms. Concretely, we adopted a flow around the circular cylinder as the simple model and discussed the difference of the unsteady structure by comparing the data reconstructed from the long period of time series of C_{d} (drag coefficient). Differences of bifurcation processes caused by the physical instability from those by the numerical instability are studied. The bifurcation scenario and the chaotic behavior have been reported in some studies. In particular, Pullium et al. [9] studied the nonlinear dynamical structure of the physical variable such as the velocity component in the subsonic flow around the airfoil by using the compressible scheme. In that paper, they not only showed the bifurcation sequence but also evaluated the effect of the grid refinement and some numerical schemes. We discuss the differences of bifurcation processes in order to clarify the analogy and dissimilarity of dependence of stability on the amplitude of second- and fourth-order viscosity terms and time increment. The numerical scheme used in the present paper is expressed briefly in Section 2. Other conditions of computations such as the grid systems, boundary conditions and so on are also described in Section 2. In Section 3, the analogy and dissimilarity of dependence of stability on several parameters by comparing the bifurcation processes are discussed in detail.