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Coded aperture optimization for single pixel compressive computed tomography

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Abstract

One of the main challenges in Computed Tomography (CT) is to obtain high-quality image reconstructions by using low-cost architectures. To alleviate this problem, the compressive sensing (CS) theory has been used to develop CT architectures with a partial reduction of the detector dimension by the inclusion of coded apertures. This dimensional reduction is possible, since, CS defy the conventional imaging notion that the detector dimension must match that of the object embedding space. Therefore, CS-CT architectures with low-resolution detectors have been developed in order to reduce the number of elements in the detector array, and so, the cost. However, in CT-CS architectures the approach of reducing the dimension of the detector array to the extreme case of a single pixel detector has not yet been studied. This work aims at studying the problem of obtaining high-quality CT images from coded projections acquired by a CS-CT architecture with a single pixel detector. Coded apertures are specially designed to enhance the peak signal-to-noise ratio of the reconstructed images. Simulation results show that the reconstructed images using the single pixel architecture gain up to 7.31 dB in average when random coded apertures are used and 14 dB in average using the designed codes with respect to a sparse view angle CS-CT architecture with full detector array, and a CS-CT architecture with modulated X-ray beam.

Keywords: Computed tomography, single pixel, compressive sensing, coded aperture optimization.

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1. Introduction

Computed tomography (CT) imaging is a non-invasive technique that has had a tremendous impact in medicine and industrial applications. This impact is due to, CT techniques estimate the internal structure of an object from a set of projections captured by a CT scanner. Conventional CT scanners are composed of an X-ray source and a detector array that rotates simultaneously around the object [1, 2]. In each view angle, the source produces an X-ray beam that passes through the object which is attenuated to a greater or lesser extent depending on the density of the object. The attenuated X-ray is captured by a detector array located on the opposite side of the source. In traditional CT reconstruction algorithms the quality of the reconstructed images depends on the number of projections acquired in each view angle, i.e. the reconstruction quality improves in proportion to the total number of elements in the detector. However, the implementation cost increases exponentially in function of the detector resolution. Therefore, low-cost architectures that attain high-quality CT images are desired.

In order to reduce the required size, complexity, and cost of CT devices, compressive sensing (CS) theory has been used to modify the CT scanning process [3, 4]. Compressive sensing computed tomography (CS-CT) strategies combine sampling and compression into a single non-adaptive linear measurement process. Two of the most used CS-CT architectures are the view angle reduction (VAR) and the X-ray beam modulation (XRBM) which employ coded apertures [4]. The coded aperture is composed of elements that block or allow the passing of the X-ray through the object, known as block-elements and pass-elements. A sketch of these architectures is shown in Figs.1.(a)-(b), respectively. In VAR and XRBM, the compression level is estimated as a function of the number of measurements acquired, and the number of non-blocking elements in the coded aperture, respectively. Specifically, CS in CT asserts that images $\mathbf{f} \in \mathbb{R}^n$ can be recovered from a significantly undersampled set of measurements $\mathbf{p} \in \mathbb{R}^m$, where $\mathbf{p} = \mathbf{\Phi f}$ and $\mathbf{\Phi} \in \mathbb{R}^{m\times n}$ is the sensing matrix with $m \ll n$. To correctly recover $\mathbf{f}$ from $\mathbf{p}$, two conditions need to be satisfied [3], [5]: first, the image $\mathbf{f}$ needs to be sparse in some basis $\mathbf{\Psi} \in \mathbb{R}^{n\times n}$, such that $\mathbf{x} = \mathbf{\Psi}\boldsymbol{\theta}$ can be approximated by a linear combination of only S elements of $\boldsymbol{\theta} \in \mathbb{R}^n$, with $S \ll n$. The second condition is that the sensing matrix $\mathbf{\Phi}$ follows an appropriate randomized sampling scheme. Taking into account the CS theory, it is possible to estimate high-dimensional objects by using low-dimensional detectors, i.e., CS theory defies the conventional imaging notion that the detector dimension must match that of the object embedding space. Therefore, to successfully exploit the CS theory, compressive ar-

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