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Stable evaluations of fractional derivative of the Müntz−Legendre polynomials and application to fractional differential equations

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Abstract

The aim of this paper is to present efficient and stable methods to compute Caputo fractional derivative (CFD) of the Müntz−Legendre polynomials based on three−term recurrence relations and Gauss−Jacobi quadrature rules. This approach with collocation method at Chebyshev− Gauss− Lobatto points has applied for solving linear and nonlinear fractional multi−order differential equations (FDEs) described in Caputo sense. The main characteristic of spectral collocation method is that the problems reduce to linear or nonlinear systems of algebraic equations. In this work, for the first time, we present the new rates of convergence for projection error which are more accurate than the rate presented by Shen and Wang in [35]. Moreover, we present convergence rate for spectral collocation method for linear FDEs with initial value on a finite interval and endpoint singularities. Also, we propse an error analysis for Jacobi-Gauss type quatrature and present a way to accelerate the convergence rate for singular integrands applied in this paper. Finally, the stability and applicability of the numerical approach and convergence analysis is demonstrated by some numerical examples.

Keywords: the Müntz−Legendre polynomials, Caputo fractional derivative, nonlinear fractional multi−order differential equations, spectral collocation method

1. Introduction

Fractional order dynamics arise in various areas of science and engineering and have found many applications in fluid−dynamics, control processing, astronautics, signal processing, robotics and economics [4, 7, 20, 23, 29, 38, 42]. In recent years, it has been demonstrated that the dynamics of many systems can be described more precisely by using fractional differential equations (FDEs). FDEs are described by various definitions of fractional derivative, such as Riemann−Liouville derivatives (RLFD) and Caputo fractional derivatives (CFD). In the current paper, we focus on FDEs with the CFD as a fractional derivative. Because of the applications of FDEs, immense efforts have been done to find the solutions of these problems and it has been an important topic for researchers [5, 10, 44, 45]. Since we cannot find analytical solutions of most FDEs except for special cases, the approximation methods must be used to approximate the solutions. Recently, some authors have paid attention to introduce some numerical schemes that approximate the solutions of FDEs, such as Adomian decomposition method [16, 27, 41], homotopy analysis method [8, 28], variational iteration method [43], spectral methods [10, 33, 36] and wavelet method [21, 32]. The main contribution of numerical methods is applying the operational matrices of the Riemann−Liouville fractional integration or Caputo fractional derivative for basis functions. As a global collocation method the class of spectral methods are more applicable and powerful tools used extensively for finding the numerical solution of FDEs. The collocation points are chosen based on accurate quadrature rules and the basis functions are typically Legendre or Chebyshev polynomials. Exponential convergence of the method and ease of applying are two prominent features which have

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