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High order integration factor methods for systems with inhomogeneous boundary conditions

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Abstract

Due to the high order spatial derivatives and stiff reactions, severe temporal stability constraints on the time step are generally required when developing numerical methods for solving high order partial differential equations. Implicit integration method (IIF) method along with its compact form (cIIF), which treats spatial derivatives exactly and reaction terms implicitly, provides excellent stability properties with good efficiency by decoupling the treatment of reaction and spatial derivatives. One major challenge for IIF is storage and calculation of the potential dense exponential matrices of the sparse discretization matrices resulted from the linear differential operators. The compact representation for IIF (cIIF) was introduced to save the computational cost and storage for this purpose. Another challenge is finding the matrix of high order space discretization, especially near the boundaries. In this paper, we extend IIF method to high order discretization for spatial derivatives through an example of reaction diffusion equation with fourth order accuracy, while the computational cost and storage are similar to the general second order cIIF method. The method can also be efficiently applied to deal with other types of partial differential equations with both homogeneous and inhomogeneous boundary conditions. Direct numerical simulations demonstrate the efficiency and accuracy of the approach.

Key words. Integration factor method, compact representation.

1 Introduction

Let Ω be an open rectangular domain in $\mathbb{R}^d$ and a final time $T > 0$. In this paper, we consider solving a system of reaction-diffusion equations:

$$\begin{cases} \frac{\partial u}{\partial t} = D\Delta u + f(u), & \mathbf{x} \in \Omega, \quad t \in (0, T), \\ u|_{t=0} = u_0, & \mathbf{x} \in \Omega, \end{cases} \tag{1}$$

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