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Empirical likelihood-based inference in generalized random coefficient autoregressive model with conditional moment restrictions

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ABSTRACT

This paper concentrates on the parameter estimating strategy for the generalized random coefficient autoregressive (GRCA) model in the presence of the auxiliary information. We propose a weighted least squares estimate for the model parameters and empirical likelihood (EL) based weights are obtained through using these auxiliary information. The asymptotic distribution of our proposed estimator is normal distribution and the asymptotic variance is reduced compared to the least square (LS) estimator. Therefore, our method yields more efficient estimates. We also carry out some simulation experiments to assess the performance of the suggested estimator and illustrate the usefulness of this method through the analysis of a real time series data sets.

1. Introduction

Hwang and Basawa [1] defined the following generalized random coefficient autoregressive model:

$$Y_t = \Phi_t^\tau Y(t-1) + \varepsilon_t,\ t = 0, 1, 2, \ldots, \tag{1.1}$$

where "τ" denotes the transpose of matrix, $\Phi_t = (\Phi_{t1}, \ldots, \Phi_{tp})^\tau$ is a random coefficient vector, $Y(t-1) = (Y_{t-1}, \ldots, Y_{t-p})^\tau$, $E\binom{\Phi_t}{\varepsilon_t} = \binom{\phi}{0}$ and $Var\binom{\Phi_t}{\varepsilon_t} = \begin{pmatrix} V_\phi & \sigma_{\Phi\varepsilon} \\ \sigma^\tau_{\Phi\varepsilon} & \sigma^2_\varepsilon \end{pmatrix}$.

Random coefficient time series models have been frequently used to describe the dynamical systems with the random disturbances in economics and biology (see, for example, Nicholls and Quinn [2] and Tong [3]). Estimating problem for the RCAR model is now well developed. For the RCAR model with independent error sequence, Nicholls and Quinn propose to estimate the model parameters by maximum likelihood (ML) and LS methods (for instance, see Nicholls and Quinn [2], Nicholls and Quinn [4] and Quinn and Nicholls [5]), Araveeporn [6] proposes to estimate the model parameters through the Bayesian method, and Berkes et al. [7] investigate the estimation problems of the nonstationary RCAR model parameters using the quasi-likelihood method. Moreover, for the RCAR model with martingale difference errors, Zhao et al. [8] propose to estimate the model parameters by the LS method. In the recent literature, some nonparametric methods have also been

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applied to the GRCA models to study the estimating and testing problems (see Zhao and Wang [9] and Zhao et al. [10]). In this paper, we consider how to use the EL method to estimate ϕ combining with the auxiliary information.

Often, when we make statistical inferences, we also know some other information in addition to the sample information that we have used. For example, the moments of the population distribution are known, or we know the relation between the population variance and the population mean. We assume that the auxiliary information can be systematically represented as

$$E(g(Y_t,\ldots,Y_{t-p};\theta_0)|Y(t-1))=0 \tag{1.2}$$

for each $t = 0,1,2,\ldots,$ where $\theta_0 \in R^d$, $g(Y_t,\ldots,Y_{t-p};\theta_0) \in R^r$ and $r \geq d$. For notational simplicity, denote $g(Y_t,\ldots,Y_{t-p};\theta)$ by $g_t(\theta)$. Here, θ can be different from ϕ and a broad class of information can be represented by $g_t(\theta)$, e.g. the conditional moments or conditional distribution of Y_t (see Hansen [11]). Because we use more information, the efficiency of the statistical inference can be increased (see Isaki [12], Kuk and Mak [13] and Rao et al. [14]). Xu et al. [15] indicate that auxiliary information can be expressed as moment restrictions, and consider the distribution estimation with moment restrictions for missing data. Tang and Leng [16] consider how to incorporate auxiliary information by using the moment restrictions to improve quantile regression. But so far, the statisticians have paid little attention to how to apply the auxiliary information to the statistical analysis for time series data. We denote the auxiliary information by the conditional moment restrictions and consider the statistical inference of GRCA model with the auxiliary information. Since the conditional moment restrictions is a generalization of moment restrictions, it can denote a wider range of information classes.

Owen [17,18] first generalized the parametric likelihood to the EL. Owen [19] and Kolaczyk [20] then applied this method to the regression problems. As a nonparametric statistical method, it has many important features. For instance, the orientation and shape of the confidence region is automatically determined by the data, its limiting properties are similar to those of parametric likelihood and it is a robust statistical method due to its nonparametric feature. In view of these attractive properties, various authors have also considered the EL inference of some other statistical models (see Chan and Ling [21], Guggenbergera and Smith [22], Zhao et al. [10] and Li et al. [23]). Moreover, to improve the estimate, some statisticians begin to investigate how to use as much as possible sample information in the statistical inference process (see Chen and Qin [24], Zhang [25] and Tang and Leng [26]).

In this paper, we concentrate on the parameter estimating strategy for the GRCA model in the presence of the auxiliary information. We obtain a more efficient estimator compared to the LS estimator. Some simulation studies also demonstrate its efficiency gain.

Section 2 introduces the estimating method and the main theoretical results. Some simulation experiments are carried out in Section 3. Section 4 provides a real data analysis. The proof of the main theorems is left to Section 5.

We use the symbols "$\xrightarrow{d}$", "a.s." and "$\xrightarrow{p}$" to denote convergence in distribution, almost surely convergence and convergence in probability, respectively. N(0,1) denotes standard normal distribution. The symbol $A \otimes B$ denotes the Kronecker product of matrices A and B, I denotes the identity matrix, the Euclidean norm of the matrix is denoted by $\|\cdot\|$ and $A > 0$ implies that A is positive definite.

2. Methodology and main results

Below, we investigate how to use the EL method (see Qin and Lawless [27]) to estimate ϕ based on the auxiliary information (1.2).

For convenience, we rewrite model (1.1): let $U_t = (\varepsilon_t, 0, 0, \ldots, 0)^\tau$ be $p \times 1$ vectors, $\tilde{\Phi}_{tj} = \Phi_{tj} - \phi_j, j = 1,\ldots,p,$

$$B=\begin{pmatrix}\phi_1 & \phi_2 & \cdots & \phi_{p-1} & \phi_p\\ 1 & 0 & \cdots & 0 & 0\\ 0 & 1 & \cdots & 0 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & 1 & 0\end{pmatrix}_{p\times p},\quad C_t=\begin{pmatrix}\tilde{\Phi}_{t1} & \tilde{\Phi}_{t2} & \cdots & \tilde{\Phi}_{tp}\\ 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & 0\end{pmatrix}_{p\times p}.$$

Then (1.1) becomes

$$Y_t = (B + C_t)Y(t-1) + U_t. \tag{2.1}$$

Throughout the paper, we make the following assumptions.

($\mathbf{A_1}$) $E(C_t \otimes C_t) + (B \otimes B)$ has all of its eigenvalues less than 1 in modulus.

Hwang and Basawa [1] indicate that ($\mathbf{A_1}$) implies that $\{Y_t, t \geq 1\}$ forms a stationary and ergodic sequence. Further, by the ergodic theorem, we can assume that the following conditions are satisfied.

($\mathbf{A_2}$) $E(g_t(\theta_0)) = 0$ for a certain θ_0, $\Sigma(\theta) = E(g_t(\theta)g_t^\tau(\theta)) > 0$ at θ_0. There exist a neighborhood $\mathfrak{A}$ of θ_0 and an integrable function $\tilde{W}(x)$ such that, in $\mathfrak{A}$, $\partial g(x,\theta)/\partial\theta$ is continuous, $\|\partial g(x,\theta)/\partial\theta\| \leq \tilde{W}(x)$, $\|g(x,\theta)\|^3 \leq \tilde{W}(x)$, and the rank of $E(\partial g_t(\theta)/\partial\theta)$ is d.