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The Cauchy problem for $u_t = \Delta u + |\nabla u|^q$, large-time behaviour

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Abstract

The nonlinear partial differential equation in the title is typified mathematically as a viscous Hamilton–Jacobi equation. It arises in the study of the growth of surfaces, and in that context is known as the generalized deterministic KPZ equation. Considering the Cauchy problem with initial data that are merely supposed to be bounded and continuous, results on the temporal decay and large-time behaviour of solutions are presented. Corresponding results for the heat equation serve as benchmarks.
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Résumé

L'équation non-linéaire aux dérivées partielles apparaissant dans le titre est du type mathématique Hamilton–Jacobi visqueuse. Elle intervient dans l'étude de la croissance des surfaces, et dans ce contexte, sous le nom de l'équation KPZ, déterministe, généralisée. Considérant le problème de Cauchy avec donnée initiale supposée, seulement, bornée et continue. Des résultats sur la décroissance temporelle, ainsi que le comportement de la solution quand le temps tend vers l'infini sont presentés. Des résultats correspondants pour l'équation de la chaleur servent de référence.
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1. Introduction

This paper is concerned with the following problem where

$$q > 0,$$

$n \geqslant 1$, and Q denotes the half-space:

$$Q := \mathbb{R}^n \times \mathbb{R}_+ \quad \text{with } \mathbb{R}_+ := (0, \infty).$$

Problem P. Solve the equation,

$$\frac{\partial u}{\partial t} = \Delta u + |\nabla u|^q \qquad \text{for } (x,t) \in Q, \tag{1}$$

where t denotes time and the operators on the right-hand side of the equation denote the standard differential operators with respect to the spatial variable $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, subject to the initial condition:

$$u(x,0) = u_0(x) \quad \text{for } x \in \mathbb{R}^n. \tag{2}$$

Eq. (1) is typified as a viscous Hamilton–Jacobi equation. It has been proposed as an appropriate model for surface growth by ballistic deposition, and specifically for vapour deposition and the sputter deposition of thin films of aluminium and rare earth metals. In this context it is known as the generalized deterministic KPZ equation [25,28–30,39].

About the initial data in the above problem, we shall assume that

$$u_0 \in C(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n). \tag{3}$$

Under this assumption [22], Problem P is well-posed, i.e.,

Lemma 1. *Problem P admits a unique solution $u \in C^{2,1}(Q) \cap C(\overline{Q}) \cap L^\infty(Q)$. Furthermore, given two such solutions $u^{(1)}$ and $u^{(2)}$ with initial-data functions $u_0^{(1)}$ and $u_0^{(2)}$ respectively,*

$$\left|u^{(1)} - u^{(2)}\right|(x,t) \leqslant \left\|u_0^{(1)} - u_0^{(2)}\right\|_{L^\infty(\mathbb{R}^n)} \quad \textit{for all } (x,t) \in Q,$$

with strict inequality if $q \geqslant 1$ and $u_0^{(1)} \not\equiv u_0^{(2)}$.

A leitmotif throughout the extant study of Problem P [1,2,5–20,22,31–33,35,36] has been the degree of similarity between solutions of Problem P and those of the Cauchy problem for the (linear) heat equation, i.e.,

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