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Convergence properties of certain refinable quasi-interpolatory operators ☆

Laura Gori [a], Elisabetta Santi [b,*]

[a] *Dipartimento di Metodi e Modelli Matematici, Via A. Scarpa, 00161 Roma, Italy*
[b] *Dipartimento di Ing. Meccanica, Energetica e Gestionale, Piazza E. Pontieri, 67040 Roio Poggio (AQ), Italy*

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Abstract

The convergence of a class of operators, both refinable and quasi-interpolatory, is analyzed and some examples are provided.

Keywords: Refinable functions; Multiresolution analysis; Refinable operators; Quasi-interpolatory operators; Total positivity

1. Introduction

The importance of linear operators in approximation theory is well known, among them a prominent role is played by quasi-interpolatory operators. Most of these latter are based on the use of polynomial or spline bases and provide local tools for practical and effective approximation of functions or discrete data [9,12–15].

On the other hand, refinability, which is at the base of subdivision schemes as well of wavelets, represents a starting point for the construction of approximation methods, which, as the most recent literature shows, enable one to obtain a variety of useful application, ranging from the solution of partial differential equations to the reproduction of curves and surfaces.

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* Corresponding author.
E-mail address: esanti@ing.univaq.it (E. Santi).

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We recall that a refinable function is the solution to two scale refinement equations of the form [2]:

$$\varphi(x) = \sum_{i\in\mathbb{Z}} a_i \varphi(2x - i).$$

The class of refinable functions we are interested in, has been introduced in [3,5] and applied, for example, in [4,10]; we will denote in the following these functions by GP refinable functions.

The idea at the base of this and other papers of the authors [7–9,11] is to join the previous items and construct particular classes of quasi-interpolatory refinable operators.

Some results in this sense are presented in [7,8] where the total positivity of the refinable function employed in the construction of certain positive operators of Marsden–Schoenberg type affords the possibility to efficiently reproduce functions preserving their shape.

More general quasi-interpolatory refinable operators will be analyzed in the present paper. They apply to a large class of functions, involve function evaluations and reproduce appropriate classes of polynomials.

The paper is organized as follows: in Section 2 we give some preliminaries on GP refinable functions; in Section 3 we introduce the quoted quasi-interpolatory operators whose convergence property is proved in Section 4. Finally, in Section 5 we present some numerical results.

2. Preliminaries

The GP refinable functions introduced in [3,5] have compact support in $[0, n+1]$, and are identified in terms of their masks, that is of the vector $a = \{a_j\}_{j\in\mathbb{Z}}$. The entries of a depend explicitly on the length of the support and on a certain number of real parameters. For the sake of simplicity, we consider here, the masks subset depending on only one real parameter $h \geqslant n \geqslant 2$, an assumption by no means restrictive. Such masks are given by:

$$a_k = \frac{1}{2^h}\left[\binom{n+1}{k} + 4\left(2^{h-n} - 1\right)\binom{n-1}{k-1}\right], \quad k = 0, 1, \ldots, n+1. \tag{1}$$

If $h = n$ we get the mask of the B-spline of n degree.

The refinable functions GP enjoy many properties useful in the applications: they are centrally symmetric, i.e. $\varphi(x) = \varphi(n+1-x)$, totally positive (TP; for the definition see [1, p. 134]), belong to $C^{n-2}(\mathbb{R})$ and there results:

$$\sum_{k\in\mathbb{Z}} \varphi(x-k) = 1, \quad x \in \mathbb{R}, \qquad \int_{\mathbb{R}} \varphi(x) = 1.$$

Moreover, they have order of polynomial exactness $d = n - 1$, i.e.

$$x^{\ell-1} = \sum_{k\in\mathbb{Z}} \beta_{0,k}^{(\ell)} \varphi(x-k), \quad \ell = 1, 2, \ldots, d, \tag{2}$$

where the coefficients $\{\beta_{0,k}^{(l)}\}_{k\in\mathbb{Z}}$ are given in [11]. Finally, any GP refinable function generates a multiresolution analysis (MRA) on $\mathbb{R}$ and a MRA on a finite interval I can be constructed using the refinable B-bases obtained as we specify below.