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Multi time scale differential equations for simulating frequency modulated signals

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Abstract

Radio-frequency (RF) circuits produce quasiperiodic signals with widely separated time scales. In the case of autonomous time scales, frequency modulation occurs in addition to amplitude modulation. A multidimensional signal model yields an efficient numerical simulation by computations via a multirate partial differential algebraic equation (MPDAE) with periodic boundary conditions. We present a time domain method for these systems, which integrates along characteristic curves and thus is consistent with the inherent information transport. Moreover, we propose a special choice for additional boundary conditions, which are necessary to determine local frequencies. Test results confirm that the constructed techniques compute efficiently frequency modulated quasiperiodic signals in RF applications.

Keywords: Circuit simulation; Frequency modulation; Multirate partial differential algebraic equation; Characteristic curves; Method of characteristics

1. Introduction

The numerical simulation of electrical circuits is based on a network approach, which typically yields systems of *differential algebraic equations* (*DAEs*) [2]. These systems describe the transient behaviour of all node voltages and some branch currents. In RF applications, circuits often include signals with largely differing time scales. Thus integrating the circuit's equations in time domain demands an enormous

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computational work, since the fastest rate limits the integration step size, whereas the slowest rate mostly decides the total time interval of the simulation.

A multidimensional signal model permits an alternative strategy by decoupling the separate time scales. Consequently, the DAE model of the circuit is transformed into a PDAE model, the *multirate partial differential algebraic equation* (*MPDAE*). Corresponding multiperiodic solutions yield the desired DAE solutions. Brachtendorf et al. [1] successfully applied this PDAE approach in frequency domain by a generalisation of harmonic balance. An analysis of the PDAE system exhibits an information transport along characteristic curves [8]. This structure enables a method of characteristics to compute multiperiodic solutions. The technique is efficient and robust for simulating MPDAE models, where the time scales are driven by input signals, see [9], and thus purely amplitude modulated responses with constant frequencies arise.

If autonomous time scales occur in addition to driven rates, then the signals may be frequency modulated, too. Narayan and Roychowdhury [7] generalised the multidimensional model to this case and introduced a *warped MPDAE* system. In contrast to the purely driven situation, an unknown local frequency function arises in addition to the multiperiodic solution. Consequently, an extra condition is required to determine uniquely the involved functions. Thus the multidimensional model is more complicated in the case of autonomous time scales.

In this paper, we tailor the method of characteristics to the warped MPDAE model. Thereby, just two time scales are considered, i.e., a forced and an autonomous rate, since modifications to several time scales are straightforward. In addition, we suggest a specific phase condition to fix the local frequency function. This condition produces additional boundary conditions in the method of characteristics. Furthermore, we develop a homotopy method, which shall secure the convergence of corresponding Newton iterations. The constructed method of characteristics is used to solve two classical test examples in circuit simulation, which represent systems of ordinary differential equations (ODEs). These ODE systems already indicate the appearance of autonomous time scales.

The paper is organised as follows. We outline the multidimensional model in Section 2. Then the corresponding warped MPDAE model is introduced. Analysing the information transport in the system, we construct a method of characteristics for the numerical solution in Section 4. Finally, test results using a Van der Pol and a Colpitt oscillator are illustrated in Sections 5 and 6, respectively.

2. Multidimensional signal model

The purpose of the multidimensional model is to represent efficiently signals including widely separated time scales. As a simple instance, we consider the amplitude modulated signal

$$v(t) = \left[1 + \frac{1}{2}\sin\left(\frac{2\pi}{T_1}t\right)\right]\sin\left(\frac{2\pi}{T_2}t\right), \tag{1}$$

with $T_1 \gg T_2$. Consequently, many time steps are required to resolve all oscillations of the fast rate T_2 during one slower rate T_1. However, we are able to change from the time-dependent signal (1) to a multidimensional representation, where each time scale is described by its own variable. Accordingly, we obtain the function

$$\hat{v}(t_1, t_2) = \left[1 + \frac{1}{2}\sin\left(\frac{2\pi}{T_1}t_1\right)\right]\sin\left(\frac{2\pi}{T_2}t_2\right), \tag{2}$$