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A charge oriented mixed multirate method for a special class of index-1 network equations in chip design ☆

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Abstract

Multirate methods make use of latency that occurs in electrical circuits to simulate more efficiently the transient behaviour of networks: different stepsizes are used for subcircuits according to the different levels of activity. As modelling is usually done by applying modified nodal analysis (MNA), the network equations are given by coupled systems of stiff differential-algebraic equations. Following the idea of mixed multirate for ordinary differential equations, a ROW-based 2-level multirate method is developed for index-1 DAEs arising in circuit simulation. To obtain order conditions, P-series are generalised to MDA-series for partitioned DAE systems.

Keywords: Circuit simulation; Partitioned systems; Differential–algebraic equations; Mixed multirate ROW schemes; P-series theory; MDA-series theory

1. Introduction

In full chip design it has to be verified whether the network design coincides with the functional demands. To do so, modified nodal analysis (MNA) is commonly used in industrial applications to generate

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automatically network model equations from designer's drafts: Kirchhoff's current and voltage laws, together with characteristic equations for each basic element based on a charge oriented description of MOS-transistors, lead to stiff differential–algebraic equations (DAEs) of the following form:

$$\begin{aligned} A \cdot \dot{z} &= f(x) \\ 0 &= z - q(x) \end{aligned} \quad \text{on } t \in [t_0, t_{\text{end}}],\ x(t_0) = x_0, \tag{1}$$

with $x \in \mathbb{R}^n$ denoting the n unknown node potentials and $f(x) \in \mathbb{R}^n$ the currents produced by static elements. The incidence matrix $A \in \{-1, 0, 1\}^{n\times m}$ describes the network's topology related to charge storing elements (capacitances) and associates charge flow $\dot{z} = \mathrm{d}q(x)/\mathrm{d}t$ caused by these elements to the static currents $f(x)$ at each node.

Electrical networks often consist of subcircuits which show largely differing levels of activity, i.e., the inner signals of some parts are characterized by a high level of activity while others tend to change quite slowly. In terms of the mathematical model the network equations comprise of systems running on different time scales. The basic idea of *Multirate methods* is to prevent parts to be integrated more often than necessary to guarantee given error tolerances. Therefor latency is exploited to reduce computational costs.

To take advantage of the multirate feature, the network model equations have to be split in an appropriate way. Since dynamic network elements are said to react slowly or fast we can suppose that two or more nodes connected by such an element have the same level of activity at each time, i.e., regarding the whole network there is no coupling between the latent and active part through capacities. Thus the network equations (1) can be split into an active (subindex a) and latent (subindex l) part that are linked only by the static currents f_l and f_a via the coupling node potentials x_a and x_l:

$$\begin{aligned} A_l \cdot \dot{z_l} &= f_l(x_l, x_a), \qquad & A_a \cdot \dot{z_a} &= f_a(x_l, x_a), \\ 0 &= z_l - q_l(x_l), & 0 &= z_a - q_a(x_a). \end{aligned} \tag{2}$$

In the following we will assume that both networks are regular, i.e., they fulfill the following special index 1 conditions:

$$\begin{aligned} &A_l \cdot \partial q_l/\partial x_l \text{ is smooth and regular along the solution } x_l(t), \\ &A_a \cdot \partial q_a/\partial x_a \text{ is smooth and regular along the solution } x_a(t). \end{aligned} \tag{3}$$

We will show in this paper how the multirate idea for ordinary differential equations can be transfered to differential–algebraic equations of type (2) and (3).

The paper is organised as follows: Starting from multirate schemes for ODE systems recapitulated in Section 2, a mixed multirate method for the coupled system (2) of index-1 DAEs is introduced in Section 3. Its order conditions are derived by generalising P-series to MDA-series theory in Section 4. Details for MDAE23, an implementation of an embedded scheme with order 3(2), conclude this paper.

2. Multirate schemes for ODE systems

Before we state and investigate a multirate method for the coupled system (2) that treats both parts with different stepsizes, we take a closer look at multirate schemes for coupled ODEs:

$$\dot{y}_L = f_L(y_L, y_A), \qquad y_L(t_0) = y_{L,0}, \tag{4a}$$

$$\dot{y}_A = f_A(y_L, y_A), \qquad y_A(t_0) = y_{A,0}. \tag{4b}$$