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Note

On T-set of T-colorings

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Abstract

Let $\mathscr{G}$ be the collection of sets T such that the greedy algorithm obtains the optimal T-span of K_n for all $n \geqslant 1$, $\mathscr{E}$ the collection of sets T such that $sp_T(G) = sp_T(K_{\chi(G)})$ is true for all graphs G. Many families in $\mathscr{G}$ or $\mathscr{E}$ have been discovered. We know that r-initial set and k multiple of s set are all in $\mathscr{G} \cap \mathscr{E}$. Liu ($T$-colorings of graphs, Discrete Math. 10 (1992), 203–212) extended k multiple of s set by union with another set S'. In this paper, we continue to study the T-set in $\mathscr{G}$ and $\mathscr{E}$, extending some T-sets by the similar way.

Keywords: T-coloring; T-graph; Span; Recursive clique

1. Introduction

T-colorings were introduced by Hale [1] in connection with the channel assignment problem in communications. In this problem, there are some transmitters in a region. We wish to assign to each transmitter a frequency and avoid the interference. The interference occurs when the difference of channels used by the transmitters falls in the given interference set T.

Given a set T of non-negative integers and T contains 0, a T-coloring of a simple graph G is a function f from $V(G)$ to the set of non-negative integers such that if $\{u, v\} \in E(G)$ then $|f(u) - f(v)| \notin T$. The T-span of a T-coloring f, denoted by $sp_T(f)$, is

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$\max\{|f(u) - f(v)|, u, v \in V(G)\}$. The T-span of G, denoted by $sp_T(G)$, is the minimum T-span over all T-colorings of G.

Let K_n be the complete graph with n vertices. For a given T, the greedy T-algorithm on K_n colors vertices of K_n sequentially. At each step, it uses the smallest possible number that will not violate the definition of a T-coloring. But the greedy algorithm does not always provide the optimal T-span. Let $\mathscr{G}$ be the collection of sets T, such that the greedy algorithm introduced above obtains the optimal T-span of K_n for all $n \geqslant 1$. On the other hand, It is well known that $sp_T(G) \leqslant sp_T(K_{\chi(G)})$ for any T and G, where $\chi(G)$ is the chromatic number of G. So it is interesting to check when the equality holds. We define $\mathscr{E}$ to be the collection of sets T, such that $sp_T(G) = sp_T(K_{\chi(G)})$ is true for all graphs G.

Given T, the T-graph denoted by G_T was defined by Liu [2] as follows:

$$V(G_T) = \{0, 1, 2, \ldots\}, \quad \{a, b\} \in E(G_T) \Longleftrightarrow |a - b| \notin T.$$

The T-graph of order n, denoted by G_T^n is the subgraph of G_T induced by the first n vertices. In G_T, the recursive clique of size i, denoted by RK_i, is the clique with vertex set defined by $V(RK_1) = \{0\}$, and for $m \geqslant 2$, $V(RK_m) = V(RK_{m-1}) \cup \{x\}$ where x is the vertex with the smallest index adjacent to all vertices of $V(RK_{m-1})$.

Throughout this paper, let Z^+ denote the set of positive integers. For any two integers a and b with $a \leqslant b$, let $[a, b]$ denote the set $\{a, a + 1, a + 2, \ldots, b\}$. Many families in $\mathscr{G}$ or in $\mathscr{E}$ have been discovered. We know that r-initial set and k multiple of s set are all in $\mathscr{G} \cap \mathscr{E}$. Liu [2] extended k multiple of s set by union with another set S'. In Section 2, we list some previous results. In Section 3, we extend the T-sets in Section 2 with a similar way.

The following characterizations of $\mathscr{G}$ and $\mathscr{E}$, provided by Liu [3], are very useful.

Theorem 1.1 (*Liu [3]*). *For any T and any positive integer m, $sp_T(K_m) = n - 1$ if and only if n is the minimum number such that $\omega(G_T^n) = m$.*

Theorem 1.2 (*Liu [3]*). *Given T, $T \in \mathscr{G}$ if and only if for all $n \in Z^+$, the maximum recursive clique in G_T^n is a maximum clique.*

Theorem 1.3 (*Liu [3]*). *For any T, $T \in \mathscr{E}$ if and only if $\omega(G_T^n) = \chi(G_T^n)$ for all $n \in Z^+$.*

2. Previous results

In this section, we list here the previous results that will be extended in Section 3.

Theorem 2.1 (*Tesman [4]*). *If $T = \{0\} \cup [s, l]$, where $s, l \in Z^+$, then $T \in \mathscr{G}$. Furthermore, if $m = ks + b$ with $k \in Z^+ \cup \{0\}$ and $1 \leqslant b \leqslant s$, then $sp_T(K_m) = k(l + s) + b - 1$.*

Theorem 2.2 (*Liu [3]*). *If $T = \{0\} \cup [s, l]$, where $s, l \in Z^+$, $T \in \mathscr{E}$ if and only if $l = ms$ for some $m \in Z^+$.*