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Note

The relation of matching with inverse degree of a graph☆

Zhongfu Zhang[a, b], Jianxun Zhang[c], Xinzhong Lu[b]

[a] *College of Mathematics and Information Science, Northwest Normal University, Lanzhou 730070, P.R. China*
[b] *Institute of Applied Mathematics, Lanzhou Jiaotong University, Lanzhou 730070, P.R. China*
[c] *Department of Mathematics, Ningbo University, Ningbo, Zhejiang 315211, P.R. China*

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Abstract

In this paper, we give out a counterexample of the Graffiti's conjecture (583), and get the best bounds of $I(T) + \alpha'(T)$, where T denotes a tree, $I(T)$ denotes the inverse degree of T and $\alpha'(T)$ matching of T.

Keywords: Graph; Matching; Inverse degree; Counterexample; Graffiti's conjecture

1. Introduction and counterexample

Let $p = |V(G)|$, the degree sequence of G be $d_1, d_2, \ldots, d_p$, then the inverse

$$I(G) = \sum_{i=1}^{P} \frac{1}{d_i}, \quad (d_1, d_2, \ldots, d_p, d_i > 0, \; i = 1, 2, \ldots, p).$$

In 1989, Graffiti gave a conjecture (conjecture 583) [3] for tree: size-(*matching* + 1) $\leqslant$ *inverse degree*, where size is the number of edges in *T*(tree); matching $\alpha'(T)$ is the edge-independent number.

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E-mail address: z.zhongfu@163.com (Z. Zhang).

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This conjecture is false. Construct the counterexample as following:

Let $T_0 = K_{1,4}$.

Let

$$V(T_k) = V(T_{k-1}) \cup \{u_0, u_1, u_2, u_3\}.$$

$$E(T_k) = E(T_{k-1}) \cup \{u_0v_0, u_0u_1, u_0u_2, u_0u_3\},$$

where $v_0 \in V(T_{k-1})$, $d_{T_{k-1}}(v_0) = 1$. Then

$$p_k = |V(T_k)| = 5 + 4k. \quad \alpha'(T_k) = k + 1. \quad I(T_k) + \alpha'(T_k) = \frac{15p_k + 9}{16}.$$

If $p_k > 9$, it is easy to see $I(T_k) + \alpha'(T_k) < p_k$.

All terminologies and symbols unexplained here can be found in [1,2,4].

2. The main results

Graffiti's conjecture is false, but it gives another question: what is the bound of $\alpha'(T) + I(T)$?

Let F be a set of tree and satisfy

(1) $K_{1,4} \in F$,
(2) For any $T \in F, T \cup K_{1,3} + uv \in F$, where $d_T(u) = 1, d_{K_{1,3}}(v) = 3$, and $V(T) \cap V(K_{1,3}) = \phi$.

Let

$$F' = \{T_k,\ k = 0, 1, 2, \ldots\}, \text{where } T_0 = P_4 : \ V(T_k) = V(T_{k-1}) \cup \{u_1, u_2\},$$

$$E(T_k) = E(T_{k-1}) \cup \{v'u_1, u_1u_2\}, \quad u_1, u_2 \notin V(T_{k-1}), \quad v' \in V(T_{k-1}),$$

$$d_{T_{k-1}}(v') = k + 1 = \tfrac{1}{2}|V(T_{k-1})|$$

then

$$\alpha'(T_k) + I(T_k) = \frac{5}{4}|V(T_k)| - \frac{1}{2} + \frac{2}{|V(T_k)|}.$$

Theorem 1. *Let T be a tree of order p $(p \geqslant 5)$, let T be any tree of order $p+1$, if $T = K_{1,p}$, then*

$$\frac{15p + 9}{16} \leqslant \alpha'(T_k) + I(T_k) \leqslant \frac{5}{4}p - \frac{1}{2} + \frac{2}{p}$$

and the right holds equality if and only if $T \in F'$.

Proof. If $2 \leqslant p \leqslant 5$, it is easy to check that theorem is true.